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LANCHESTER COMBAT MODELS WITH SUPPRESSIVE FIRE AND/OR UNIT DISINTEGRATION

Russell Reddoch

Naval Postgraduate School
Monterey, California

March 1973

NAVAL POSTGRADUATE SCHOOL

Monterey, California

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SUPPRESSIVE FIRE AND/OR UNIT DISINTEGRATION

by

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A quick test for determining the winner of a Lanchester model combat for the case of homogeneous attrition equations is developed.

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Lanchester Combat Models with
Suppressive Fire and/or Unit Disintegration

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I. INTRODUCTION

A. GENERAL

The ability to predict the outcome of a future battle or war has always been desirable. Diplomatic policy is greatly influenced by a country's perceived view of the outcome of a conflict should one start. Military strategy is also influenced by predictions about the possible success of different strategic/tactical plans. An accurate prediction capability also allows a wise choice of force composition and size to insure victory in potential conflicts without tying up excessive resources to provide a safety margin.

Among the general techniques for making such predictions which have developed are Simulation/War Gaming and Lanchester Equation models. The former seek to capture "all significant factors" with consequent stochastic results and complexity. The latter depend upon multiple events and the law of large numbers to allow using deterministic differential equations. These two approaches are often complementary - Lanchester Equations serving to narrow down the parameter range and Simulation/War Gaming supporting and refining the Lanchester results. The refinement comes from considering factors which were ignored in the Lanchester model. Introducing some of these additional factors into the differential equations will improve the utility of the predictions they provide.

B. NEW FACTORS

This paper investigates the impact of incorporation of suppressive fire and a nonconstant combat effectiveness for individual elements into Lanchester equation models of combat. These factors are known to be of some importance in infantry ground combat and are not considered in the classical models of combat.

When two units engage in combat a large portion of their fire is not, primarily, intended to kill enemy elements, but is intended to stop them from firing upon the friendly unit. This effort to gain "Fire Superiority" is important since it normally insures winning the battle. A previous thesis [Ref. 1] modeled fire superiority by allowing an individuals rate of fire, and hence the attrition rate coefficients, to depend upon the intensity of received fire. This paper incorporates suppressive fire by adding a suppressive capability for a unit's weapons in addition to the usual casualty-causing lethal fire capability. The developed models are then used to gain some insight into the effect of changing the relative capability of the lethal and suppressive components of a forces firepower. The method of incorporating suppressive firepower also allows the developed models to be used for many other scenarios besides ground combat.

When an infantry combat unit receives casualties its combat performance falls off faster than the proportion of

casualties would indicate. This occurs because of a decay in individual element performance as its unit receives casualties. Historically, for typical infantry combat [Ref. 2] the individual combat effectiveness has dropped to zero percent when the unit has received thirty to fifty percent casualties. This effect is incorporated by association a percentage effective figure with the number of casualties a unit has taken. This figure determines how close to the original (one hundred percent) effectiveness the unit's elements are operating as the battle progresses.

II. SUPPRESSIVE WEAPONS

A. DEFINITION

A suppressive weapon is a system which reduces own force casualties by inhibiting the action of enemy weapons. It does no damage to the enemy units; these may act with full effectiveness whenever suppression is lifted. A suppressive weapon is an active system directed against specific targets or areas. Employment of own force suppressive weapons does not effect friendly firepower.

The majority of weapons have both a suppressive and a lethal component. Such a weapon will be treated as if it was two virtual weapons, one suppressive and the other lethal. The force is equipped with these virtual weapons in a ratio reflecting the original weapon characteristics. A suppressive weapon reduces the total enemy effective force, but will be treated as if it only affects hostile casualty producing components. (Suppressive weapons are not themselves suppressed.) This restriction is often satisfied as is shown in later scenarios. It also serves to make the mathematical models more tractable by cutting off long chains of cross suppression.

B. DISCUSSION

A suppressive weapon, to be effective, must be difficult to learn to ignore. The weapon may achieve this by actively intruding into the enemy weapon's functioning as an Electronic

Counter Measures (ECM) system does, by physically stopping the enemy unit's missiles as the Phalanx anti-missile gun system is designed to do, or by being associated with a casualty producer which will inflict casualties upon the hostile unit if it attempts to continue unrestricted activity. It is this need to take protective cover from fire with a consequent reduction in own weapon effectiveness which gives most weapons a dual suppressive/lethal capability.

If opponents are well trained a weapon may be too lethal to have any suppressive effect. If the target individual believes taking cover will do no good all his effort will be directed towards destroying the weapon before it hits him. In this case one may actually achieve "negative suppression" as the enemy redoubles fire in effort to destroy weapon he can not protect himself against. This may be observed in counter-ambush doctrine [Ref. 3] where ambushee avoids taking cover in the ambusher's targeted killing zone. As the killing zone is carefully selected it is fatal for the ambushee to allow himself to be pinned down in it, hence the doctrine of prompt counterattack at all costs despite the resulting exposure. Flamethrower tanks were strong "negative suppression" weapons during World War II. When a flame tank moved into position to attack bunkers it would receive heavy fire from the bunker occupants. The bunker fire would not be suppressed by supporting tank fire which normally was sufficient to drive occupants to cover. This was because flame tanks represented

such a high threat of destruction regardless of cover taken that desperate risks were justified to destroy them.

The interaction between lethality and suppression was also experienced in Vietnam when river gunboats had their 40^{mm} Automatic Weapons replaced by 105^{mm} Howitzers. This was done because the 40^{mm} weapon had proven ineffective against the bunkered positions from which hostile elements normally fired. The 105^{mm} howitzer was able to destroy bunkers with its more powerful shell. The immediate effect of this change was, surprisingly, increased friendly force casualties. The 40^{mm} weapon was unable to penetrate bunker walls, but generated a highly visible volume of fire, and had a good lethality potential against personnel who remained at the bunker firing slits. The 105^{mm} weapon lacked the visible volume of fire and was almost as dangerous against personnel who had taken cover as it was against those who continued to fire. Since taking cover gained little, the enemy fire was not suppressed by the heavier weapon.

III. NONCONSTANT EFFECTIVENESS

A. DEFINITION

A combat unit's elements exhibit nonconstant effectiveness when the performance of individual elements depend upon the other elements of the unit. The physical destruction of (1 - K) percent of the unit reduces the unit's combat effectiveness by more than (1 - K) percent. This is because the surviving elements lose some of the support they have been receiving, and may additionally have to devote part of their effort to aid casualties, so their individual effectiveness drops too. The elements are assumed to provide no redundancy in the unit's basic capability, so a unit at K percent strength will never have a combat power greater than K percent of its original power. In general the unit's combat power will be $f(K) \cdot K$ percent of its original power where $f(K)$ is a decreasing function of K, $f(K)$ represents the fraction of full effectiveness of the individual elements at K percent of the unit strength.

B. DISCUSSION

It is extremely rare to find a combat unit that does not exhibit some non-linearity in its effectiveness. A combat unit generally has an internal command structure which means the individual elements are not completely identical. Destruction of comman elements has a greater impact on the unit than loss of other elements. Even when the

command element's functions are promptly taken over by another individual there is a break in the continuity of direction, with a consequent reduction in the unit performance.

Personnel, moreover, have some concern for their individual survival. As a unit absorbs casualties individuals become convinced that their continued fighting is doing little good and only insures their own destruction. The casualty level required to induce this feeling depends upon training, expected treatment if wounded, and how visibly the enemy is suffering, but serves as a breakpoint for any unit. (The breakpoint is also dependent upon the individuals having a way of breaking off the combat safely.)

While these effects may accumulate over time as the collapse of the French and Russian Armies in 1917 [Ref. 4] shows, generally a short recuperation and reorganization period will reestablish internal effectiveness. This ability to reestablish combat effectiveness given time to reorganize is one of the reasons that time is an important parameter of an engagement.

IV. SCENARIOS

It is useful to have some typical scenarios in mind when constructing mathematical models of combat. This makes it easier to follow the relationship between the developed equations and the actual combat.

A. INFANTRY (MODEL I)

Two small infantry units engage in combat without external supporting weapons. The location of the members of the opposing force are known and fire is reasonably well distributed. (No target is left unengaged if a firer is available to engage it.) Each side employs aimed fire and can judge the effectiveness of its fire. If an individual is suppressed (due to his sensing enemy near misses) he changes his position to a more protected one. While he remains in the more protected position he is unable to aim accurately enough to inflict casualties, however he continues firing and may still suppress enemy personnel. When a combatant ceases fire, his opponent(s) shift fire to a new target. The forces do not maneuver once engaged and the weapon parameters are constant throughout the battle.

B. INFANTRY (MODEL II)

Two small non-maneuvering infantry units engage in combat. The position of members of the enemy force is reasonably well known, so all personnel attempting to inflict casualties have definite targets to shoot at. An

assigned fraction of each force (riflemen) deliver aimed semi-automatic fire attempting to inflict casualties. The remaining fraction of each force (automatic weapons) deliver area fire attempting to suppress enemy fire. As casualties occur personnel are reassigned so the ratio of semi-automatic to automatic fire remains constant. The volume of fire delivered by the automatic weapons is so large that the suppressive effect of the individual riflemen can be neglected. The targets vulnerable area is a small enough fraction of the area receiving automatic weapons fire that casualties produced by the automatic weapons can be ignored.

The division between aimed (casualty producing fire) and area (suppressive fire) would also arise if a force was only partially successful in locating targets. Those individuals who have located targets direct aimed fire at them, using semi-automatic fire for accuracy. Those unable to locate a target fire upon area enemy occupied in an attempt to suppress enemy fire. The fraction of a force which is able to locate exact targets remains constant (a function of terrain) as individuals gain and lose track of hostile personnel during the battle.

C. AIR-ECM-GROUND (MODEL I)

An aviation force consisting of attack and ECM aircraft engages an air defense missile system. The aviation force is attempting to eliminate the air defense system, the air defense system to inflict losses upon the aviation force.

The defense force may have units capable of deceiving attack aircraft's weapons delivery/navigation systems so some aircraft will waste their ordnance (be suppressed) on false targets. The ECM aircraft are able to prevent a fixed number of missile sites from shooting effectively. Neither side can tell which type of unit he is attacking and both are equally vulnerable when attacked so force composition stays essentially in fixed ratio during battle. The weapon system parameters remain constant during the engagement.

D. SHIPS VS AIRCRAFT-MISSILE ATTACK (MODEL I)

A unit of ships equipped with anti-aircraft missiles and an anti-missile gun system (Phalanx) are engaged by an aviation force equipped with anti-shipping missiles and ECM systems. The anti-missile gun system effectively suppresses some aircraft attacks by destroying inbound missiles before they can reach the ships. The ECM equipment suppresses some anti-aircraft fire by jamming some outbound missiles. A sufficient supply of missiles is available to both sides so neither side is supply limited during the engagement. (If this is not the case a diverted/intercepted missile represents a permanent reduction in combat potential as no round would be available to fire when suppression was "lifted." This would make suppressive weapons both suppressive and lethal weapons in the model construction.)

E. COUNTERBATTERY (MODEL V)

Two artillery units engage in counterbattery fire, each attempting to silence the other. The exact location of opposing guns is not known so that fire is directed into area enemy occupies. The gun crews will seek cover if enemy fire lands too close to them, and will reman the gun when fire is lifted. A direct hit is required to destroy the gun, while shell fragments will drive the crew to cover, so the suppressive fire effect is large compared to the lethal fire effect.

V. MODEL I CONSTANT, AIMED, AIMED

A. GENERAL

This model applies when both sides are able to identify targets and deliver aimed lethal and aimed suppressive fire at them. In addition, the effectiveness of the individual combat elements of each side remains constant despite casualties to other elements. The unit therefore exhibits a combat effectiveness which is a linear function of the number of survivors. This means that the battle follows the general Lanchester "square law" combat equations modified to account for the effects of fire suppression.

Each force is divided into two fractions. One fraction inflicts casualties and the other fraction suppresses enemy fire. It is assumed to be impossible to identify the enemy divisions so fire may not be concentrated upon one of the parts of the enemy force, therefore the percentage assigned to each mission remains constant throughout the battle. This is usually due to the fact that the two missions are actually being performed by one weapon and the fractional split is a property of the weapon.

B. MODEL

The differential equations for aimed lethal fire with aimed suppressive fire may be developed from the classical Lanchester equations for aimed fire. The number of elements involved in firing suppression is determined, $(1 - \alpha)X$, from

this the number of hostile elements suppressed is calculated, $(1 - \alpha)\beta bX$, and the active hostile strength determined by subtracting this from the actual enemy strength, $Y - (1 - \alpha)\beta bX$. The fraction of enemy strength delivering lethal fire, γ, times the active enemy strength yields the effective enemy force, $\gamma(Y - (1 - \alpha)\beta bX)$. The effective enemy strength is used instead of numerical strength in the classical Lanchester equations for aimed fire. The revised equations therefore are:

$$\frac{dx}{dt} = -a\gamma(Y - (1 - \alpha)\beta bX)$$

$$\frac{dy}{dt} = -b\alpha(X - (1 - \gamma)\delta aY)$$

where

$(-\frac{dx}{dt})$	is the rate of attrition of the X force
$(-\frac{dy}{dt})$	is the rate of attrition of the Y force
a,b	are the attrition rate coefficients for Y,X respectively, the number of casualties inflicted per unit time per firing unit
α,γ	are the fractions of X,Y force delivering lethal fire
β,δ	are the suppressive effectiveness ratios for X,Y so βb and δa are the number of hostile units which an element assigned to suppression suppresses.

If the elements firing suppressive fire were themselves suppressed the equations would take the form:

$$\frac{dx}{dt} = -a\gamma(Y - (1 - \alpha)\beta b(X \underbrace{- (1 - \gamma)\delta a(Y - \ldots}_{\text{neglected terms}}))$$

since b, a are normally less than one and neglected terms involve squared, cubic and higher powers the error introduced is likely to be small even if the scenario indicates that suppressive component can itself be suppressed.

If it is desired to introduce suppressive fire into an aimed fire attrition model of form

$$\frac{dx}{dt} = -a^*Y \qquad \text{and} \qquad \frac{dy}{dt} = -b^*X$$

without changing the casualty producing ability of the forces this may be done by setting

$$a = \frac{a^*}{\gamma} \qquad \text{and} \qquad b = \frac{b^*}{\alpha}$$

The two units then have the same lethal firepower as before coming from the fraction of force assigned to casualty fire. If β, δ equal zero so suppressive fractions do nothing the forces will follow the same time history as they did in the model without suppression. This makes it possible to look at how suppression effects battles without concern over the fact that only a fraction of strength is now delivering

lethal fire. It is important to note that the existence of suppressive fire will naturally cause the two force history plots to diverge from each other.

C. SOLUTION

It is possible to solve this model's differential equations analytically. The solution follows standard techniques and is straightforward therefore it is only outlined. Letting

$$K = a\gamma, \qquad L = ab\beta(1 - \alpha)\gamma, \qquad M = b\alpha, \qquad N = ab\delta(1 - \gamma)\alpha$$

so equations take the more easily manipulated form:

$$(1) \quad \frac{dx}{dt} = -KY + LX \quad \text{and} \quad (2) \quad \frac{dy}{dt} = -MX + NY.$$

Differentiating (1)

$$\frac{d^2X}{dt^2} = -K(-MX + NY) + L\,\frac{dx}{dt}$$

solving (1) for Y and substituting

$$\frac{d^2X}{dt^2} = (N + L)\,\frac{dx}{dt} + (KM - LN)X$$

giving the following differential equation to solve.

$$X'' - (N + L)X' - (KM - LN)X = 0$$

with I.C. $X(t = 0) = X_o$

$$X'(t = 0) = -KY_o + LX_o$$

The general solution takes the form

$$X(t) = \exp(\frac{N + L}{2} t)\ (A\cosh \theta t + B\sinh \theta t)$$

$$\theta = \sqrt{\frac{(N - L)^2}{2} + KM}$$

and applying the initial conditions and substituting yields

$$X(t) = X_o\ E(t)(\cosh \theta t - \frac{1}{\theta}(a\gamma\frac{X_o}{Y_o} + \frac{1}{2}ab(\alpha(1 - \alpha)\delta - \gamma(1 - \alpha)\beta)\sinh \theta t)$$

$$Y(t) = Y_o\ E(t)(\cosh \theta t - \frac{1}{\theta}(b\frac{Y_o}{X_o}\alpha + \frac{1}{2}ab(\gamma(1 - \alpha)\beta - \alpha(1 - \gamma)\delta)\sinh \theta t)$$

$$E(t) = \exp(\frac{1}{2}ab(\ (1 - \alpha)\gamma\beta + (1 - \gamma)\alpha\delta)t)$$

$$\theta = \sqrt{(\frac{ab(\delta(1 - \gamma)\alpha - \beta(1 - \alpha)\gamma}{2})^2 + ab\gamma\alpha}$$

It is important to note that these equations are only valid until one side or the other is totally suppressed (at time ts) and consequently producing zero enemy casualties

where

Y(ts) = (1 − [illegible]) [illegible] (ts)

or

X(ts) = ([illegible]) [illegible] Y(ts).

If β,δ are not zero this will always occur before either side has been reduced to zero force level. Thus one can not find the duration of combat by solving for the time when one side has zero force. It is necessary to find the time at which one side is totally suppressed by substituting so the time solution into the equation for the relationship of the two forces at time ts. Since the attrition rate equations are homogeneous and of the same degree it is possible to use the test of Chapter X to determine the winner and hence which of the two equations to substitute into. Observe that E(t) is a common factor in X(t), Y(t) substitution yields equation of form:

$$G(X_o, Y_o) = \tanh ts$$

which can be solved easily to get time ts at which the loser is totally suppressed.

From the time total suppression occurs the forces follow equations of the form:

$$\frac{dx}{dt} = 0 \qquad\qquad \frac{dx}{dt} = -a\gamma(Y - (1 - \alpha)\beta bX)$$

OR

$$\frac{dy}{dt} = -b\alpha(X - (1 - \gamma)\delta aY) \qquad \frac{dy}{dt} = 0$$

so that the time, tf, at which the battle ends is given by:

$$\ln\left(\frac{X(ts)}{X(ts) - (1 - \gamma)\delta aY(ts)}\right) = (1 - \gamma)\delta ab\alpha(Tf - ts) \quad \text{if X wins}$$

and

$$\ln\left(\frac{Y(ts)}{Y(ts) - (1 - \alpha)\ bX(ts)}\right) = (1 - \alpha)\beta ab\gamma(tf - ts) \quad \text{if Y wins.}$$

Alternatively, a totally suppressed force may no longer be able to exert any influence on the battle. If this is true then

$$(tf - ts) = \frac{Y(ts)}{X(ts)b\alpha} \qquad \text{or} \qquad (tf - ts) = \frac{X(ts)}{Y(ts)a\gamma}$$

The graphical plots all assume the latter equations in calculating the time from total suppression to elimination of the loser's forces. This represents the shortest time in which the loser might be finished since any residual suppressive capability will reduce the victors kill rate

and hence increase the time needed to kill off the suppressed force.

The model is constructed assuming that an element is either combat effective or suppressed. It is, however, possible to use the model for a suppressive system that only partially suppresses an element's combat capability. This partial suppression weapon is replaced by a virtual weapon which totally suppresses a smaller number of enemy elements. The model is then valid until all elements are partially suppressed. For a given partial suppression factor, f, assuming Y is partially suppressed first this occurs when:

$$f\ Y(t_{psy}) = (1 - \alpha)\beta b\ X(t_{psy})$$

and the time at which the equations break down may be solved for in the same manner as ts is found. The battle then follows:

$$\frac{dx}{dt} = -a(1 - f)\ Y$$

$$\frac{dy}{dt} = -b\alpha(X - (1 - \gamma)(1 - f)\delta a\ Y)$$

a special case of the general equations where X has no suppression capability. Notice that only the winner will have his suppression of loser becoming limited. It is also

possible to start out with the loser's suppression of the winner limited in which case battle follows similar equations until the loser's forces have been reduced to the point where

$$g\,X(t_{psx}) = (1 - \gamma)\delta a\,Y(t_{psx})$$

from which time the full equations follow.

D. NUMERICAL RESULTS

A Fortran IV program was written to generate data for plots of force levels versus time. The state equations were solved every two minutes and the resulting force levels plotted in Figures 1-8. Battles 1 and 2 are classical aimed fire engagements shown for comparison with the suppressive fire battles 3-8. As one alternative to extending the equations to include suppressive fire is a "gross" model where initial suppressed forces are subtracted off, a classical aimed fire engagement run and suppressed forces then added back to give final force levels Battles 3A - 6A present plots of this "gross" model for comparison with the refined model. Battles 7-8 are suppressive fire battles where inclusion of suppressive fire has altered the outcomes. A summary of the forces, parameters, and results are provided in Table I.

It is clear from Figures 3-6 that the "gross" suppression model provides a highly erroneous estimate for the duration

of combat. It is therefore not an acceptable alternative to refining the classical equations. Examination of the battle plots shows considerable model response to parameter changes. In general increasing the fraction of both forces assigned to suppression lengthens the resulting engagement. Raising the suppressive effectiveness of both forces tends to shorten the engagement and decrease the victors casualties.

The combined effect of these two trends is quite nice because of current difficulties with estimating the correct values for attrition rates. The values obtained from firing range test data have been considerably higher than those calculated from analysis of historical battle data. The introduction of suppressive fire effects, as in going from Battle 1 to Battle 5, appears to be able to account for a large part of this discrepancy. The large firing range test values can be taken as correct for the small fraction of weapons which actually have targets, and are delivering lethal fire, the remaining weapons are delivering only suppressive fire and it is the averaging of the lethal fire over all the weapons in the classical model which leads to the difference between the two estimates.

Fire suppression thus accounts for another factor besides casualty firepower effecting the outcome of battle. It should be noted that the results of battle between X and Y change as X gains in suppressive effectiveness from Battle 6B (Y wins) through Battle 7 (Tie) to Battle 8 (X wins).

It is obvious that it is always advantageous to increase the suppressive effectiveness of a unit, if other parameters are not changed. The real world rarely allows this so it is desirable to gain some idea on the interrelation of a unit's parameters. The reproduced Fortran IV program included is designed to aid in doing this. It takes a base force, parameter set and generates sets of α-β values so that the battle will either take the same time or the winner will receive the same casualties as in the base battle. Figures 9-13 are graphs of the output from this program. The shapes taken by these tradeoff curves are very dependent on the composition of the Y forces. The regions above the curves represent improvement for the X force. If X is the winner either the battle will be shorter or X will receive fewer losses. If Y is the winner either the battle will take longer or Y will receive more casualties. It is important to note that once $(1 - \alpha)\beta bX = Y$ there is no advantage to be gained from higher β values or lower α values. (Y can only be suppressed once.) It is also important to note that the restriction of β to positive values means that a constant value solution is not always possible.

To simplify plotting curves for other formulas for the time from total suppression to end of combat the residual time calculations have been included in the program as subroutines. It may be necessary to modify statement 1500 in the main program if the residual time routines are changed.

This statement computes the minimum fraction of X force needed to eliminate Y in base time if Y is totally suppressed at time zero.

VI. MODEL II - NONCONSTANT EFFECTIVENESS - AIMED FIRE

A. GENERAL

This model applies when both sides are able to identify targets and deliver aimed lethal fire only. As each unit absorbs casualties its internal organization and morale break down. This breakdown and the diversion of resources to aid wounded results in the surviving elements operating at a lower combat efficiency than they did at the start of the battle. When casualties have become sufficiently high the unit losses all its combat capability, although a sizable fraction may still survive.

B. MODEL

The basic Lanchester equations for an aimed fire battle are:

$$\frac{dx}{dt} = -aY \qquad \text{and} \qquad \frac{dy}{dt} = -bx.$$

These equations may be modified to reflect the loss in individual combat efficiency as the units cohesion is destroyed by adding a percentage effective correction term. An idea of the way effectiveness actually decays is provided by [Ref.2], Figure 14 shows the general relationship between percent effective and percent casualties developed there. A reasonable, yet tractable fit is provided by

$$\text{percent ineffective} = (\text{percent casualties})^n$$

or

$$\text{percent effective} = 1 - k(\text{Initial Force} - \text{Present Force})^n.$$

To determine k observe that if X_0 is the strength at which a unit becomes completely ineffective then

$$0 = 1 - k(X_o - X_b) \qquad \text{so} \qquad k = \frac{1}{X_o(1 - BP)}$$

where BP is the percent strength at which the unit breaks and effectiveness equals zero. To determine the proper value for the shape factor n the curves are fitted to the results of the Fast Val Study [2] for a Break Point of seventy percent. This leads to values in the range 2.5-3.8 for a reasonable fit. So percentage effective may be expressed as

$$PE = 1 - \left(\frac{X_o - X}{X_o(1 - BP)}\right)^n.$$

Multiplying numerical strength times percentage effective yields effective strength which is then used in the classical Lanchester equations. The revised Lanchester equations thus take the form:

$$\frac{dx}{dt} = -a\left(1 - \left(\frac{Y_o - Y}{Y_o(1 - BP)}\right)^m\right)Y$$

and

$$\frac{dy}{dt} = -b(1 - (\frac{X_o - X}{X_o(1 - BP)})^n)X.$$

C. SOLUTION

It is possible to solve for the equation of state. Dividing the first equation by the second gives

$$\frac{dx}{dt} = \frac{a(1 - k_y(Y_o - Y)^m)Y}{b(1 - k_x(X_o - X)^n)X}$$

separating variables and integrating we get for integer m,n

$$b(\frac{X^2 - X_o^{\,2}}{2}) - bk_x((\frac{X_o - X)^{n+2}}{n+2} - \frac{X_o(X_o - X)^{n+1}}{n+1})$$

$$= a(\frac{Y^2 - Y_o^{\,2}}{2}) - ak_y((\frac{Y_o - Y)^{m+2}}{n+2} - \frac{Y_o(Y_o - Y)^{n+1}}{n+1}).$$

To compare this result with the classical Lanchester aimed fire equation of state: $b(X_o^{\,2} - X^2) = a(Y_o^{\,2} - Y^2)$ multiply through by minus one so the leading terms take the same form. Now note that for the region of applicability of the model $(X_o - X)^{n+2} < (X_o - X)^{n+1}X_o$ and $\frac{1}{n+2} < \frac{1}{n+1}$ so the "new" terms are always being added to the classical Lanchester terms. (Subtracted from classical terms when equations are placed in the traditional form.)

It is useful to observe that early in the engagement $X_o \sim X$ and $Y_o \sim Y$ so the "Square Law" terms dominate the state equation and $b(X_o^2 - X^2) \simeq a(Y_o^2 - Y^2)$ which compares nicely with the classical equation of state.

The time solution is quite intractable. Even fixing $n = m = 1$ yields a second order monster of the form:

$$Y'' = -a(-b(1-k_y(Y_o-Y)Y) + ak_xX_o(-b(1-k_y(Y_o-Y))Y) + ak_x2X(-1)(1-k_y(Y_o-Y))Y),$$

now $Y' = -aX = ak_xX_oX - aX^2$ so the quadratic may be solved for X. It is possible to determine that the positive root is the desired one. In the region of interest Y' is negative so use of the negative square root would lead to a negative value for X, hence

$$X = \frac{(ak_xX_o - a + ((a-ak_xX_o)^2 - 4aY')^{\frac{1}{2}})}{2a} .$$

However, even setting Breakpoint at zero percent so $k_x = \frac{1}{X_o}$, $k_y = \frac{1}{Y_o}$ leaves a differential equation of the form:

$$Y'' = AY + BY\frac{(-aY')^{\frac{1}{2}}}{a} + CY^2 + DY^2\,\frac{(-aY')^{\frac{1}{2}}}{a}$$

to solve and the differential inside the radical makes the equation completely untractable.

These difficulties lead to a solution by numerical approximation to generate the time trace of forces during a

battle. The differential equations are replaced by a pair of difference equations:

$$X(t+\Delta t) = X(t) - a(1 - k_y(Y_o - Y(t))^m Y(t))\Delta t$$

$$Y(t+\Delta t) = Y(t) - b(1 - k_x(X_o - X(t))^n X(t))\Delta t$$

where t advances in steps of size Δt instead of being a continuous variable. The equations are seen to be of the first order since the time variable has a spread of only one time step Δt.

The accuracy with which the difference equations results approximate the true solution to the differential equations depends upon the choice of the size of the time step. It is often possible to verify suitability of a time step size by comparing the approximate results to the exact solution of a solvable special case. These differential equations unfortunately could not be solved for any case so the suitability of the time step had to be verified in a more heuristic manner. The equations of Model I provide a completely solved set of complex differential equations, accordingly a Δt was found which gave agreement ± .25 between a finite difference approximation and the exact solution. This time step size was used for the numerical approximations of succeeding models. To provide some more confidence that the approximate results were reasonably close to the true results for the nonconstant aimed fire

model one approximation was done using a much smaller step size. This change had negligible effect on the force behavior over time, so it is felt the approximation is reasonably accurate.

D. NUMERICAL RESULTS

A Fortran IV program was written to produce data for time plots of force behavior during battle. Figures 15-18 show the behavior for different shapes of the effectiveness versus strength curve and an increasingly stubborn X force. Changing the shape parameters n and m has very small effect on the program results for parameter values which give a good fit to the Fast Val Curve of Figure 14. Y casualties went from 5.57 to 5.88 as the shape parameter went from 3 to 4. These losses are much smaller than the 15.28 casualties Y receives in the classical Lanchester equation model. The engagement is also shorter taking only 45 minutes compared to the classical 160 minutes. This shorter time is caused by the rapid disintegration of the losing side.

The close agreement between the classical state equations and the nonconstant state equations early in combat should be noted. It is normal procedure to rotate units to avoid the breakdown this model predicts and this rotation generally keeps the battle in the "early phase." This accounts for the reasonable agreement the classical equations have had with historical battles even though subunits nonconstant combat effectiveness is ignored. The normal course of

battle provides the subunit with chances to reorganize and evacuate casualties thereby restoring itself to full combat effectiveness.

VII. MODEL III NONCONSTANT, AIMED, AIMED

A. GENERAL

There is no reason for suppressive fire and nonconstant element effectiveness not to occur at the same time. This leads to a combined model which applies when both sides are able to identify targets and deliver lethal and suppressive fire at them. The effectiveness of the individual elements of the units are changed by casualties to other elements. When a unit's casualties become sufficiently high the unit ceases to be combat effective, the survivors break up into a noncombative rable.

B. MODEL

The properties of Models I and II may be combined. Determine the percentage effective for X and Y forces using the effectiveness formula of Model II. The number of hostile units suppressed is calculated as in Model I using force times its percentage effective for strength of friendly forces to allow for the smaller effective number of elements. The active enemy strength then equals the numerical strength minus suppressed elements. The fraction of enemy strength assigned the lethality fire mission times the active enemy strength times the percentage effective yields the effective enemy force. The effective enemy force is used instead of numerical strength in the Lanchester equations. This gives equations:

$$\frac{dx}{dt} = -a\gamma(1 - (\frac{(Y_o - Y)^m}{Y_o(1 - BP)}))(Y - (1-\alpha)\beta b(1 - (\frac{(X_o - X)}{X_o(1 - BP)})^n)X)$$

$$\frac{dy}{dt} = -b\alpha(1 - (\frac{X_o - X}{X_o(1 - BP)})^n)(X- (1-\gamma)\delta a(1 - (\frac{Y_o - Y}{Y_o(1 - BP)})^m)Y).$$

These equations may be approximated as a pair of difference equations of the first order as was done for Model II and approximate numerical solutions generated by a step by step integration.

C. NUMERICAL RESULTS

A Fortran IV program was written to produce data for time plots of force behavior during battle. Figures 19-24 show the behavior for different shapes of the effectiveness vs strength curves, and an increasingly stubborn X force. Changing the Shape Parameters n and m had small effect on the results of the battle. The number of Y casualties increased from 3.75 to 3.98 as the shape went from 3.0 to 4.0. These losses are smaller than the 5.7 casualties Y takes in the previous model. The engagement also lasts longer, taking 96.0 minutes compared to 45.0 minutes. This agrees with the previously found effect of adding suppressive fire, the battle lasts longer and winner has lower casualties. The increasingly stubborn X force was able to raise Y's losses to 6.5 at the cost of increasing its own losses, before breaking, from 12.3 to 24.8. This serves as a measure

of the cost that is paid if a force is given no alternative to continued combat.

VIII. MODEL IV CONSTANT, AIMED, AREA

A. GENERAL

This model applies when both sides are able to deliver aimed lethal fire. The suppressive component of each sides fire is not accurately directed, however, and follows the classical area fire Lanchester equations. This may occur because of the nature of the weapons system or because only part of the force is able to observe a target. Those elements without a definite target deliver area fire, attempting to suppress enemy fire. The suppressive effect of the aimed fire and the lethal effect of the area fire are small enough to be neglected. The unit effectiveness is a linear function of its surviving elements.

B. MODEL

To modify Model I for area suppressive fire instead of aimed suppressive fire it is necessary to adjust the number of suppressed elements for the new mode of fire. For area fire, following the classical results, the effect of fire depends not only upon the number of firing elements but also upon the number of enemy elements in the target area. The number of elements suppressed thus equals the product of the two forces strengths times the number of enemy units suppressed per firing unit per exposed unit. This suppression value is then substituted into the equations of Model I giving new equations:

$$\frac{dx}{dt} = -a\gamma(Y - (1 - \alpha)\beta bXY)$$

$$\frac{dy}{dt} = -b\alpha(X - (1 - \gamma)\delta aXY)$$

where the parameters are as defined for Model I with β,δ in units of $\dfrac{\text{enemy units suppressed per unit exposed}}{\text{enemy units killed}}$ to account for the area nature of the suppressive fire, where the number of units suppressed depends upon the number of targets as well as the number of firing units. Note that this means for the same initial suppressive effect the parameters β,δ will be much smaller than they were in the previous models.

C. NUMERICAL RESULTS

A Fortran IV program was written to produce data for time plots of force behavior. Figures 25-28 show the time trace of force levels for different values of the parameters. It is important to note that these values can not be directly compared with those in the earlier models because of the area nature of the suppressive effort. The considerably smaller numerical values of β,δ used here were chosen to give approximately the same initial suppression effects that were present in the earlier models. Since the suppression term decays to zero as one of the forces is driven to zero a direct equivalence is not possible.

It is rather surprising to observe that the Y force devoted to lethal fire at X (Y effectives) actually increases during the course of battle 21. The suppression is quite heavy in this battle amounting to thirtysix units suppressed at the start of the battle. In this situation it is actually possible to destroy suppressing forces with a consequent release of own forces to combat faster than one is receiving casualties.

IX. MODEL V CONSTANT, AREA, AREA

A. GENERAL

This model applies when neither side is able to locate targets and consequently delivers area lethal and suppressive fire. The effectiveness of individual elements of the units are not changed by casualties to other elements. The unit effectiveness is a linear function of its surviving elements. The battle follows the general Lanchester equations for area fire, modified to account for the effects of fire suppression.

Each force is divided into two fractions, one which inflicts casualties and the other which suppresses enemy fire. The ratio of forces assigned to these two functions remains constant throughout the engagement.

B. MODEL

The Lanchester equations for area fire are:

$$\frac{dy}{dt} = -aXY \qquad \text{and} \qquad \frac{dx}{dt} = -bYX \ .$$

As before the effective force is determined by subtracting suppressed elements and multiplying by the fraction assigned to lethal fire. This effective strength is then used in the classical equations giving:

$$\frac{dy}{dt} = -a\gamma X(Y - (1 - \alpha)\beta bXY)$$

and

$$\frac{dx}{dt} = -b\alpha Y(X - (1 - \gamma)\delta aXY)$$

where the parameters are as defined for Models I and IV with a,b the rate at which a unit of force Y,X kills exposed units of force X,Y. The equations may be simplified to

$$\frac{dy}{dt} = -a\gamma XY(1 - (1 - \alpha)\beta bX)$$

and

$$\frac{dx}{dt} = -b\alpha XY(1 - (1 - \gamma)\delta aY).$$

These equations may then be rewritten as a pair of first order difference equations and solved by numerical approximation.

C. RESULTS

It is well known that for equal parameter values the classical Lanchester equations for area fire lead to equal losses. This occurs because effective firepower depends on the product of the forces engaged and the two forces therefore have equal firepower. The introduction of suppressive fire destroys this symmetry, the larger force now receiving fewer

casualties than the smaller. This occurs because each side has exactly the same number of elements suppressed (as a consequence of area fire suppression) and the suppressed elements remaining as targets. The suppressed elements represent a higher fraction of the small force and its effective firepower is reduced more than the large force's. Thus if the initial forces are $X = 40$, $Y = 60$ and suppression is 20, Y's effective firepower is $40(60-20) = 1600$ units while X's effective firepower is $(40-20)60 = 1200$ units. This advantage is retained by the larger force throughout the battle.

Figures 29-32 represent graphs of force levels over time plotted from numerical approximation results generated by a Fortran IV program. The accuracy of the Fortran program was verified by comparing results for $\alpha = \gamma = 1$, no suppression, with the known solution to the classical area fire model. Observe that the battles have infinite duration as elimination of forces also reduces the casualty rate. The unequal losses resulting from incorporation of fire suppression can be seen. When X has taken 39 casualties in battles 23 and 24 Y has received only 34.2 and 38.7. The majority of this gain occurs early in the battle were the magnitude of the difference in effective firepower is large.

X. VICTOR PREDICTION

The Lanchester type equations developed are fairly complex. Even for Model I where an analytic solution exists it is not easy to determine the winner of a battle given a set of initial force levels and parameter values. The other models lack analytic solutions so prediction of the outcome is even harder.

If the attrition equations are homogeneous (of the same degree) it is always possible to simply determine the winner of an engagement. Let $u = Y_o/X_o$ where Y_o, X_o are the initial force levels. Partition the set of possible future force levels by a line through the origin of slope u. Now determine how force levels initially move by substituting into attrition equations, getting $X'(0)$ and $Y'(0)$ and noting that forces motion is in the direction $(X'(0),Y'(0))$. Now if

a) $Y'(0)/X'(0) = u$ the force level moves along the partition line. This means the new X,Y are such that $Y/X = u$ so $X = kX_o$, $Y = kY_o$ and it follows from the homogeneity of the attrition equations force will always remain on the partition line. The battle is a tie.

b) $Y'(0)/X'(0) > u$ (and $Y'(0) < 0$, $X'(0) < 0$) the force moves below the partition line, moreover if the force ever returns to the partition line $Y/X = u$ so $X = kX_o$, $Y = kY_o$. This means at the new point $(X,Y)Y'(t)/X'(t) = k^nY'(0)/k^nX'(0)>u$

again so force will move below partition line again. Hence future force levels are confined to the lower (right hand) side of partition line which means that X is the winner.

c) $Y'(0)/X'(0) < u$ ($Y'(0) < 0$, $X'(0) < 0$) the force moves above partition line and in the same fashion future force levels are confined to the upper (left hand) side of the partition line and Y is the winner.

It is clear that homogeneity is not a necessary condition for the technique to work since it is sufficient for $X'(X = kX_o) = f(k)X'(0)$ and $Y'(Y = kY_o) = f(k)Y'(0)$ as is the case for Model II. Intuitively the test is expected to hold far more generally since the force ratio Y/X is usually a non-decreasing (non-increasing) function of time. This means that if the test indicates a winner the force levels leave the partition line and never return. It is therefore unnecessary to worry about what would happen if one did return at some lower force levels.

Parameter values in Battles 7, 17, 21 and 25 were chosen so the test would come very close to indicating a tie. The attrition equations for Battle 7 are homogeneous of degree one and the outcome plotted in Figure 7 is a tie as predicted. The equations for Battle 17 do not satisfy the sufficient conditions and the test incorrectly predicts a victory for Y ($Y/X = 1.5$, $Y'/X' = 1.4997$). The outcome plotted in Figure 23 shows that X is actually the winner. The tests failure here appears to be due to its complete insensitivity to the Breakpoint assigned to the engaged

forces, at the start of battle percentage effective is always one hundred percent. If X's breakpoint is changed to 0.7 (equal to Y's) then Y wins the battle. The test also failed to predict the correct winner in Battle 21 (Y/X = 1.5, Y'/X' = 1.5005) predicting X wins while actually Y wins the battle as shown in Figure 27. The test did succeed in correctly predicting the outcome of Battle 25 (Y/X = 1.5, Y'/X' = 1.4998) predicts Y and Y is the actual winner. In all cases after a short period of combat (prediction indicator no longer extremely close to value for tie battle) the test started to predict the correct winner.

XI. FURTHER STUDY

It would be of value to combine Models I and V where the fraction of personnel (u,v) who have succeeded in acquiring targets deliver aimed lethal/suppressive fire and those who have not deliver area lethal/suppressive fire. The equations would look like

$$\frac{dx}{dt} = -a_1 v(Y_{eff}) - a_2(1 - v)(Y_{eff})X$$

where

$$Y_{eff} = Y - (u(1 - \quad)b_1 X + (1 - u(1 - \quad)b_2 XY).$$

Since SLA Marshall [Ref. 5] indicates that a sizable fraction of a small infantry unit has not acquired a target at any given time during an engagement this model should be of considerable value. Parameter estimation would appear to be the largest problem.

It would be useful to verify that neglecting the suppression of suppressive fire weapons is not significant. There is also a need to investigate the fact that suppressive fire effect is not a linear function of the number of elements firing. The initial fire increment is actually of much more value since initial fire drives elements to cover and additional fire only effects the degree of cover taken.

This is particularly of interest when considering fire allocation problems where existing solutions often require all fire to be, unrealistically, directed at only one enemy unit. The large suppression return from small initial fire increment is likely to yield some assignment of fire to all enemy units.

XII. CONCLUSIONS

A. SUPPRESSIVE FIRE

Fire suppression is generally considered to be an important factor in combat. Introducing it into Lanchester equations models yielded several interesting insights.

1. The attrition-rate coefficients measured from firing range tests are much higher than the attrition-rate coefficients calculated from historical battle data. It appears that a large part of this difference is due to neglecting to consider the effects of suppressive fire and poor target acquisition on the firing range derived figures.

2. The equal expected losses for symmetric parameters, a = b, in an area fire engagement of the classical equations breaks down when suppressive fire is added. The model also shows the value of protection which makes suppression more difficult even if it is unable to improve survivability.

3. The equating of ECM equipment and missile interception systems with suppressive weapons, enabling the equations developed here to be used to investigate tradeoffs in force composition.

B. NONCONSTANT EFFECTIVENESS

The fact that keeping force levels close to initial values generates a force level history (state equations) which are approximately those generated by the classical Lanchester equations for aimed fire combat. This agreement

holds up even if force levels decrease considerably from their initial value if ample time exists for force reorganization during combat. This reorganization keeps individual elements operating close to full effectiveness. Since existing force relief policies attempt to provide for this reorganization and maintain units near initial strength it is usually reasonable to ignore the effect of non-constant force element effectiveness.

C. VICTOR PREDICTION

The quick test for predicting the winner of a Lanchester combat makes it possible to investigate a wide range of parameter values without having to solve equations.

TABLE I

BATTLE PARAMETERS

BATTLE	FORCE	ATR COEF	FRAC CAS	SUPP EFF	BK PT	SHAPE	WINNER	SURVIVORS	DURATION
1 -A-	40.0 60.0	0.005 0.005	- -	- -	- -	- -	Y	44.7	160
2 -A-	40.0 60.0	0.01 0.01	- -	- -	- -	- -	Y	44.7	80
3A -AG	40.0 60.0	0.005 0.005	- -	- -	- -	- -	Y	46.7	150
3B -AA	40.0 60.0	0.005 0.005	0.5 0.5	20.0 20.0	- -	- -	Y	46.5	302+19
4A -AG	40.0 60.0	0.005 0.005	- -	- -	- -	- -	Y	53,3	108
4B -AA	40.0 60.0	0.005 0.005	0.5 0.5	100.0 100.0	- -	- -	Y	52.9	220+100
5A -AG	40.0 60.0	0.01 0.01	- -	- -	- -	- -	Y	58.7	26
5B -AA	40.0 60.0	0.01 0.01	0.5 0.5	100.0 100.0	- -	- -	Y	58.7	53+99
6A -AG	40.0 60.0	0.005 0.005	- -	- -	- -	- -	Tie	- -	
6B *AA	40.0 60.0	0.005 0.005	0.5 0.5	350.0 100.0	- -	- -	Y	46	420+98

7 -AA	40.0 60.0	0.005 0.005	0.5 0.5	433.3 100.0	- -	- -	Tie	- -	
8 -AA	40.0 60.0	0.005 0.005	0.5 0.5	500.0 100.0	- -	- -	X	36.8	227+321
9 NA-	40.0 60.0	0.005 0.005	- -	- -	0.7 0.7	3.0 3.0	Y	54.4	45
10 NA-	40.0 60.0	0.005 0.005	- -	- -	0.5 0.7	3.0 3.0	Y	51.0	80
11 NA-	40.0 60.0	0.005 0.005	- -	- -	0.3 0.7	3.0 3.0	Y	47.9	125
12 NA-	40.0 60.0	0.005 0.005	- -	- -	0.7 0.7	2.5 2.5	Y	54.6	45
13 NAA	40.0 60.0	0.005 0.005	0.5 0.5	100.0 100.0	0.7 0.7	3.0 3.0	Y	56.3	96+199
14 NAA	40.0 60.0	0.005 0.005	0.5 0.5	100.0 100.0	0.7 0.7	3.5 3.5	Y	56.1	96+199
15 NAA	40.0 60.0	0.005 0.005	0.5 0.5	100.0 100.0	0.7 0.7	4.0 4.0	Y	56.0	96+199
16 NAA	40.0 60.0	0.005 0.005	0.5 0.5	100.0 100.0	0.3 0.7	3.0 3.0	Y	53.5	204+119
17 NAA	40.0 60.0	0.005 0.005	0.5 0.5	433.3 100.0	0.5 0.7	3.0 3.0	X	31.3	296+593
18 NAA	40.0 60.0	0.005 0.005	0.5 0.5	500.0 100.0	0.7 0.7	3.0 3.0	X	36.9	204+517

19 -Aa	40.0 60.0	0.005 0.005	0.5 0.5	2.0 2.0	- -	- -	Y	47.3	336+9
20 -Aa	40.0 60.0	0.005 0.005	0.5 0.5	6.0 6.0	- -	- -	Y	57.0	368+48
21 -Aa	40.0 60.0	0.005 0.005	0.5 0.5	6.89 2.00	- -	- -	Y	33.9	612+11
22 -Aa	40.0 60.0	0.005 0.005	0.5 0.5	9.0 2.0	- -	- -	X	24.2	884+35
23 -aa	40.0 60.0	0.0005 0.0005	0.5 0.5	2.0 2.0	- -	- -	Y	21.2	524**
24 -aa	40.0 60.0	0.0005 0.0005	0.5 0.5	20.0 20.0	- -	- -	Y	25.9	456**
25 -aa	40.0 60.0	0.0005 0.0005	0.5 0.5	53.33 20.00	- -	- -	Y	14.5	776**
26 -aa	40.0 60.0	0.0005 0.0005	0.5 0.5	75.0 20.0	- -	- -	X	3.8	1530**

*Element Effectivness/ Lethal Fire/ Suppressive Fire

N= Nonconstant	A= Aimed	A= Aimed
	a= Area	a= Area
		G= Gross

** Battle with area lethal fire never ends.Indicated time to reduce effective stregth of loser to 0.5 elements.

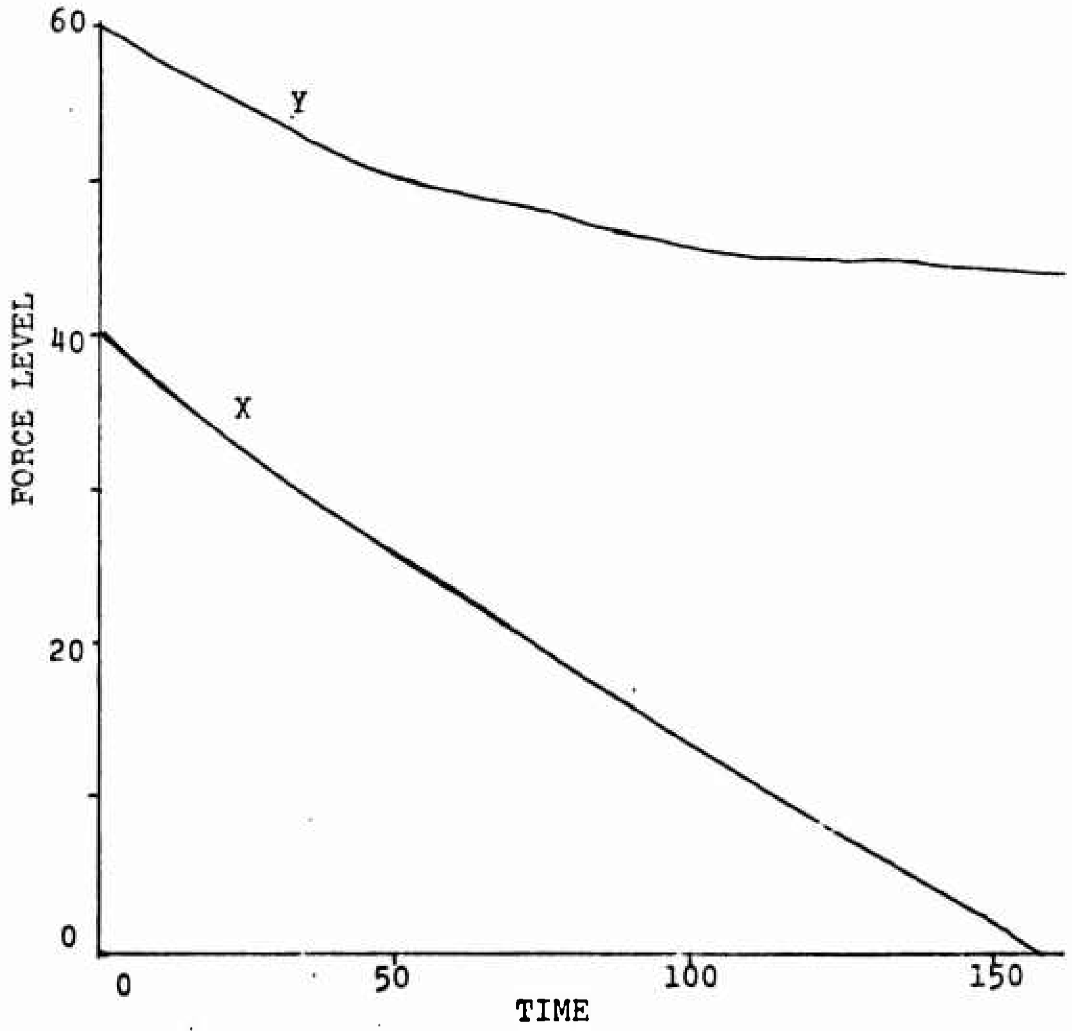

FIGURE 1

Battle 1 Classical Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	-	-	-	-
Y	60.0	0.005	-	-	-	-

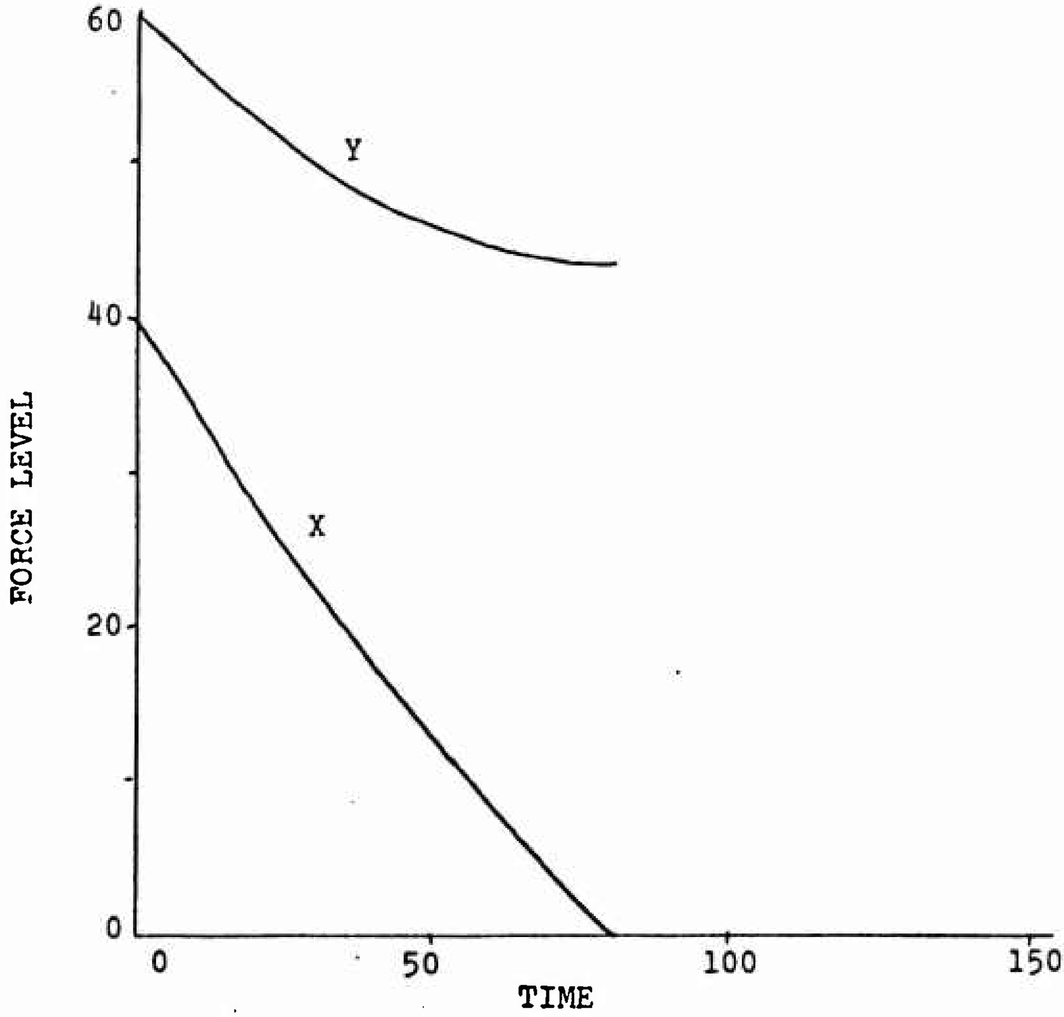

FIGURE 2

Battle 2

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	-	-	-	-	-
Y	60.0	-	-	-	-	-

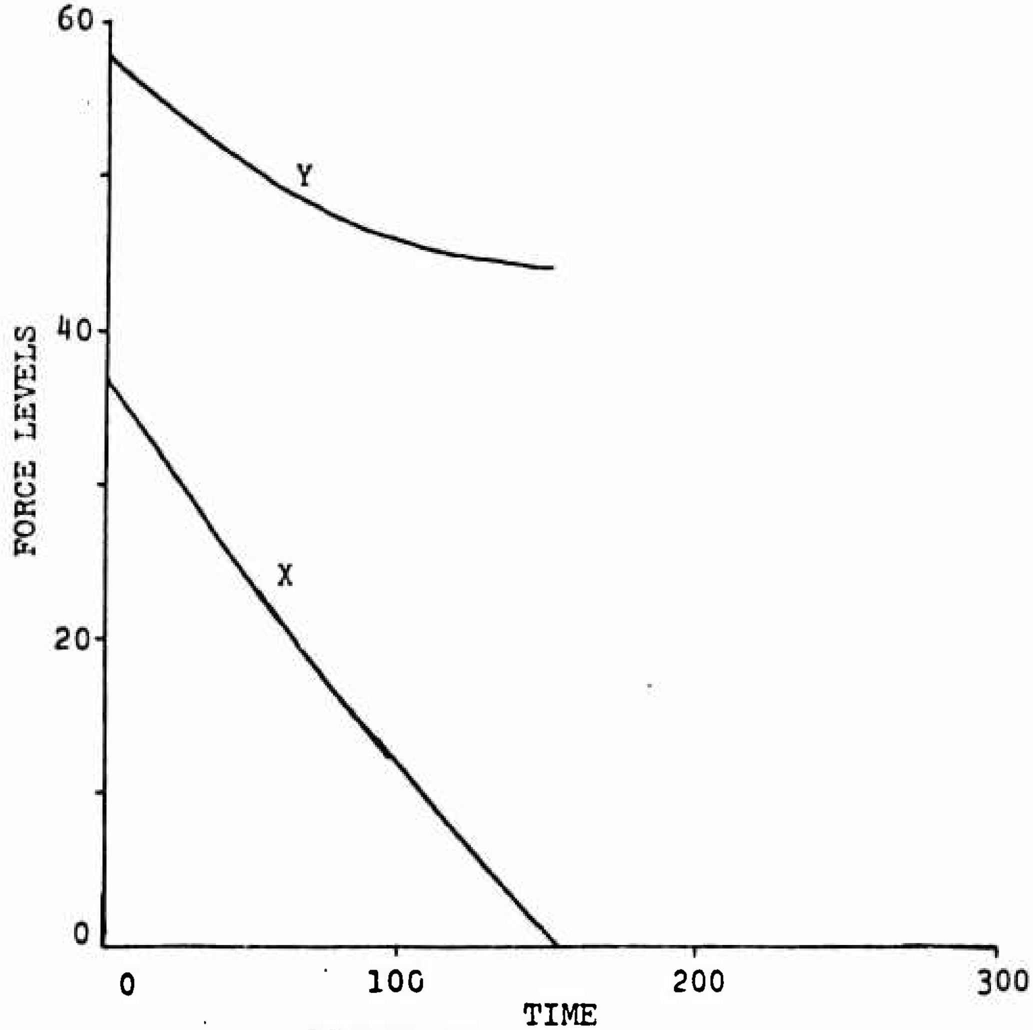

FIGURE 3A

Battle 3A - Constant, Aimed, Gross Supp

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.010	-	-	-	-
Y	60.0	0.010	-	-	-	-

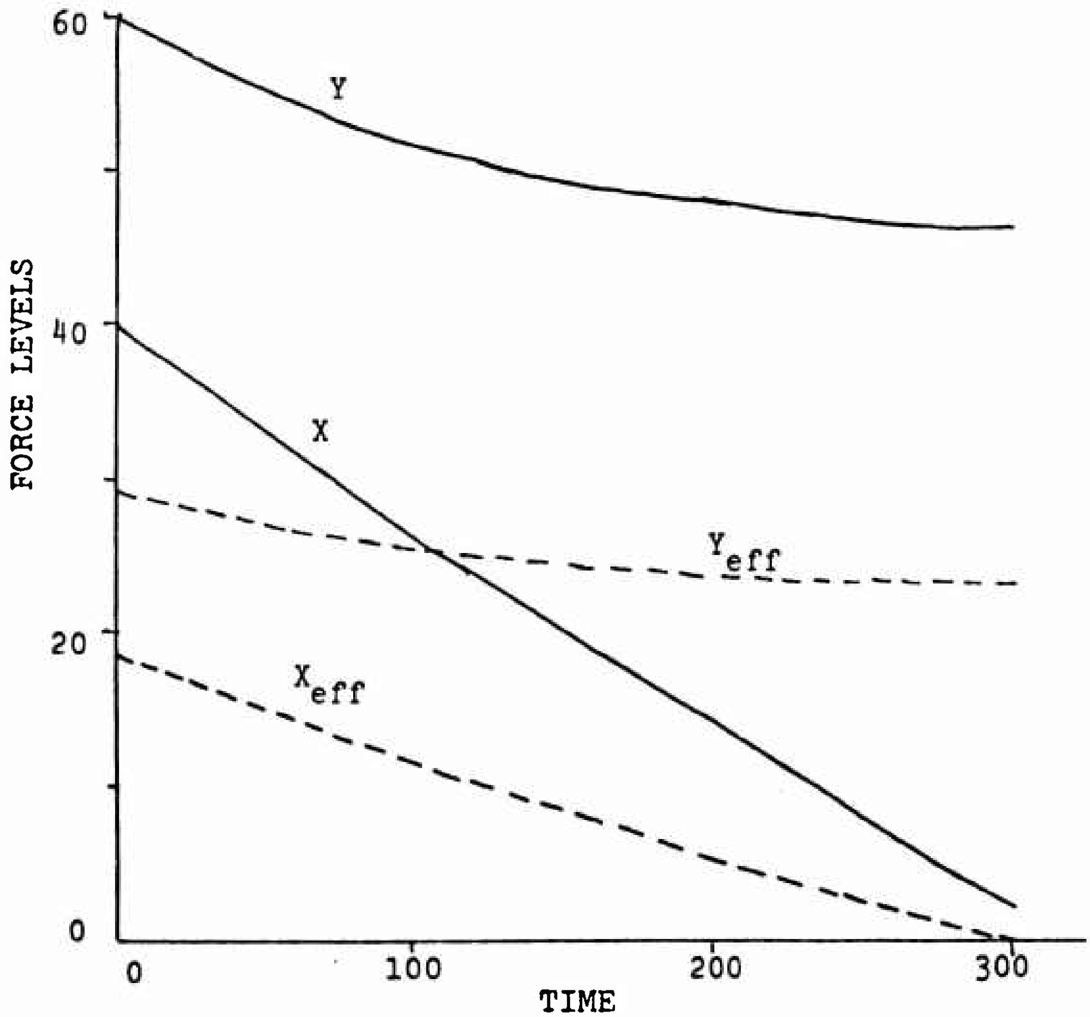

FIGURE 3B

Battle 3B Constant, Aimed, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	20.0	-	-
Y	60.0	0.005	0.5	20.0	-	-

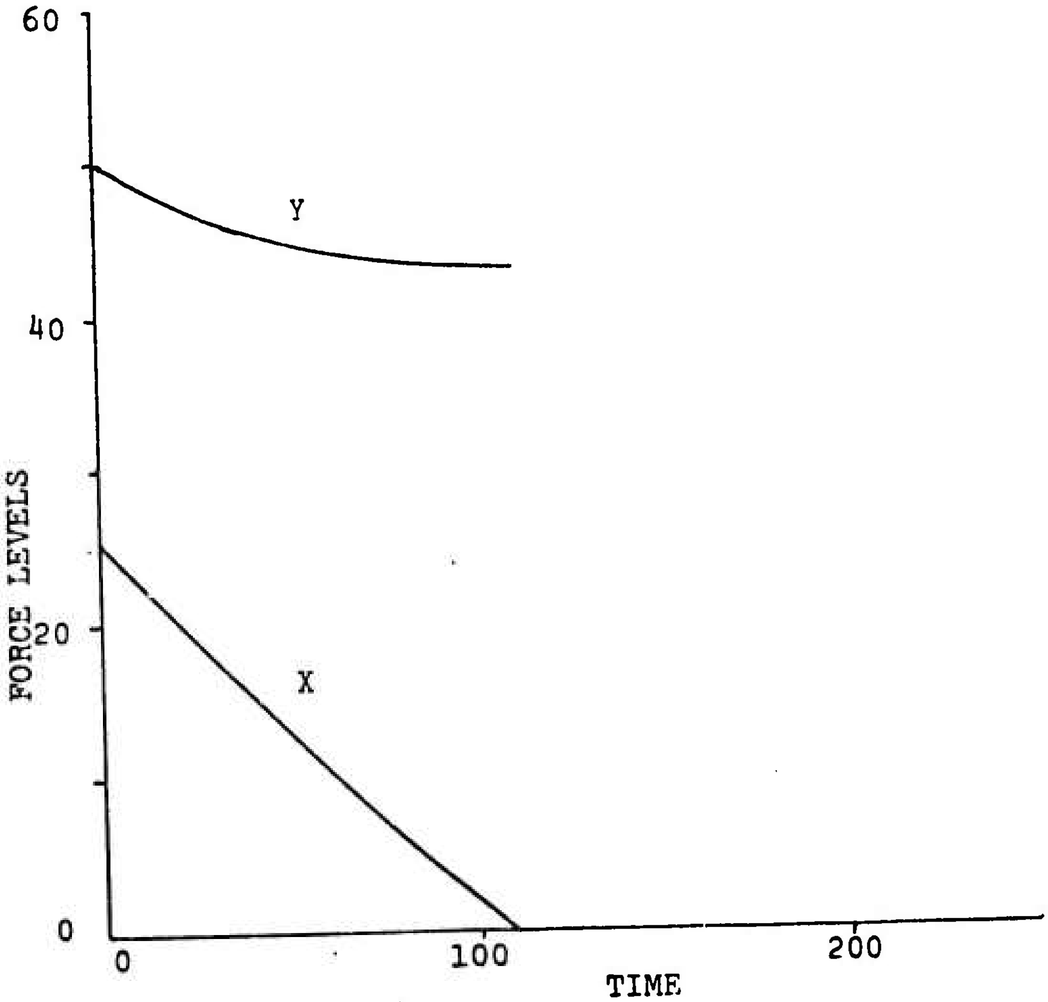

FIGURE 4A

Battle 4A Constant, Aimed, Gross Supp

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	100.0	-	-
Y	60.0	0.005	0.5	100.0	-	-

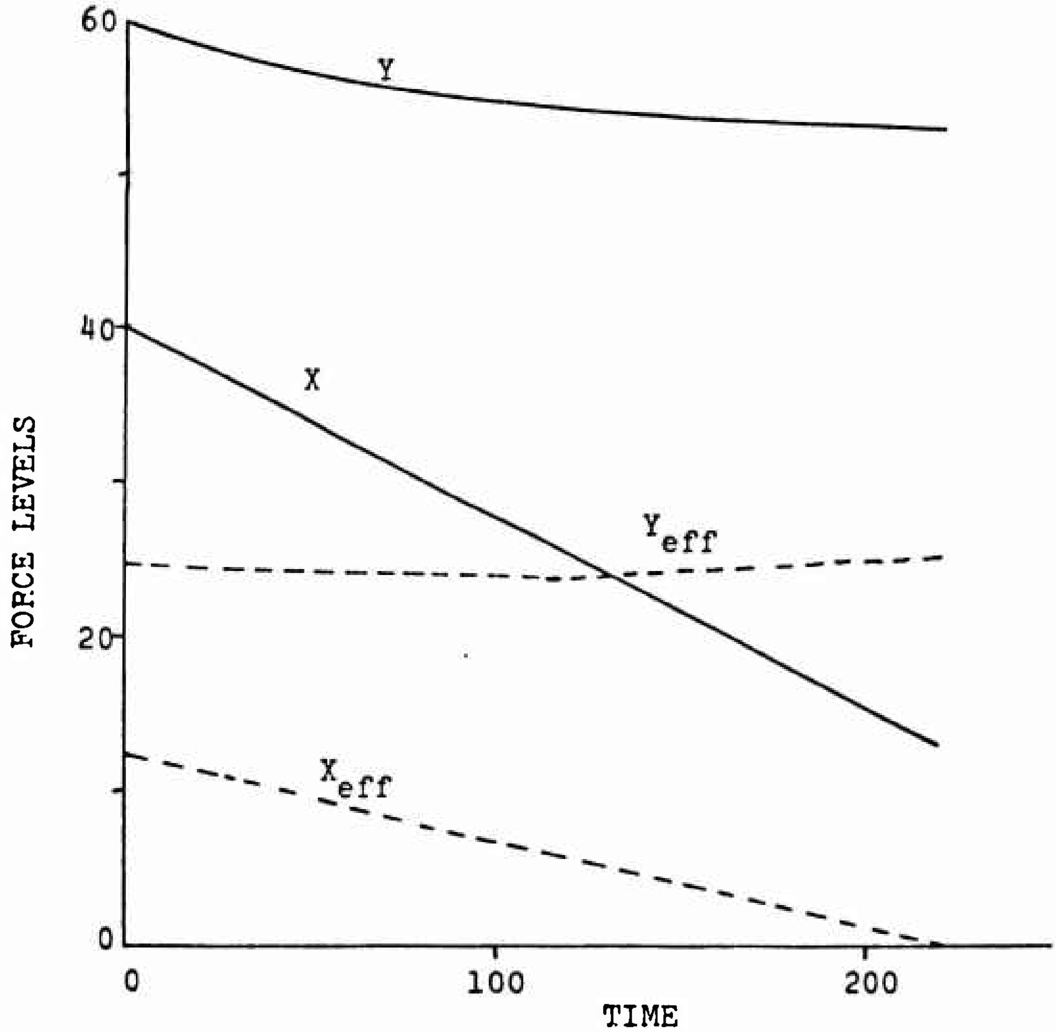

FIGURE 4B

Battle 4B Constant, Aimed, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	100.0	-	-
Y	60.0	0.005	0.5	100.0		

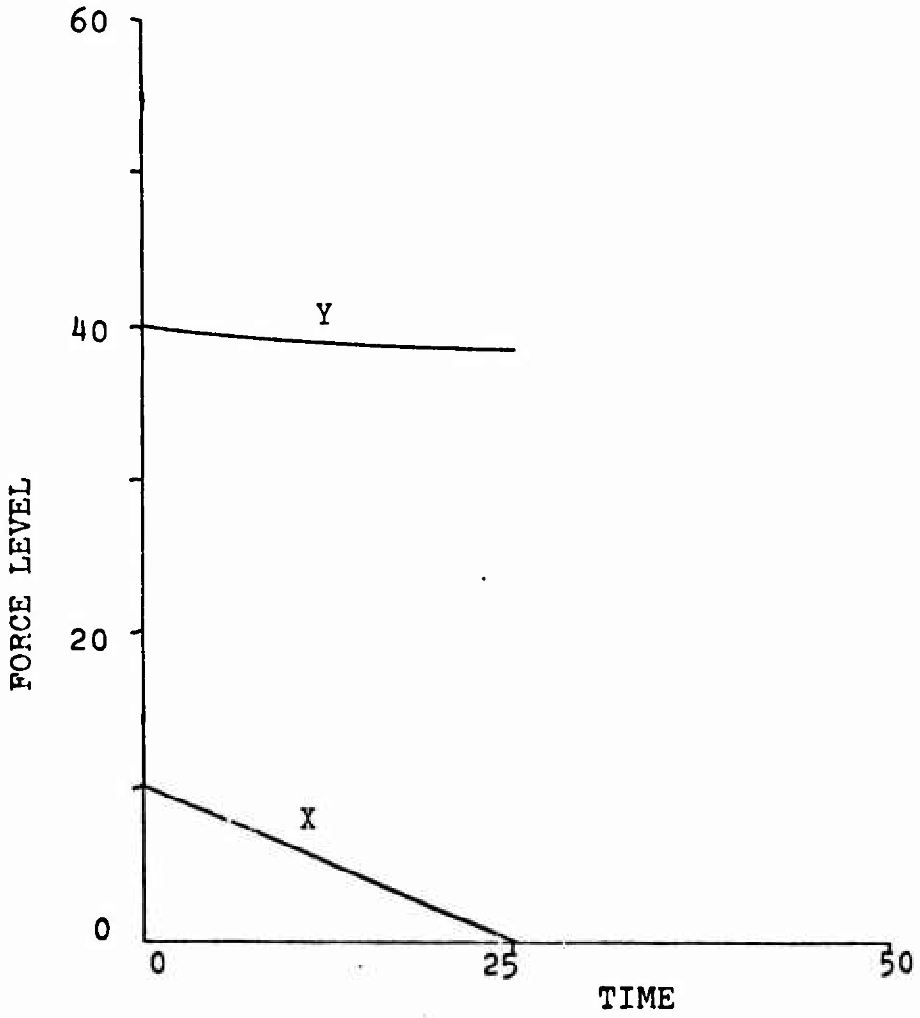

FIGURE 5A

Battle 5A Constant, Aimed, Gross Supp

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.01	0.5	100.0	-	-
Y	60.0	0.01	0.5	100.0	-	-

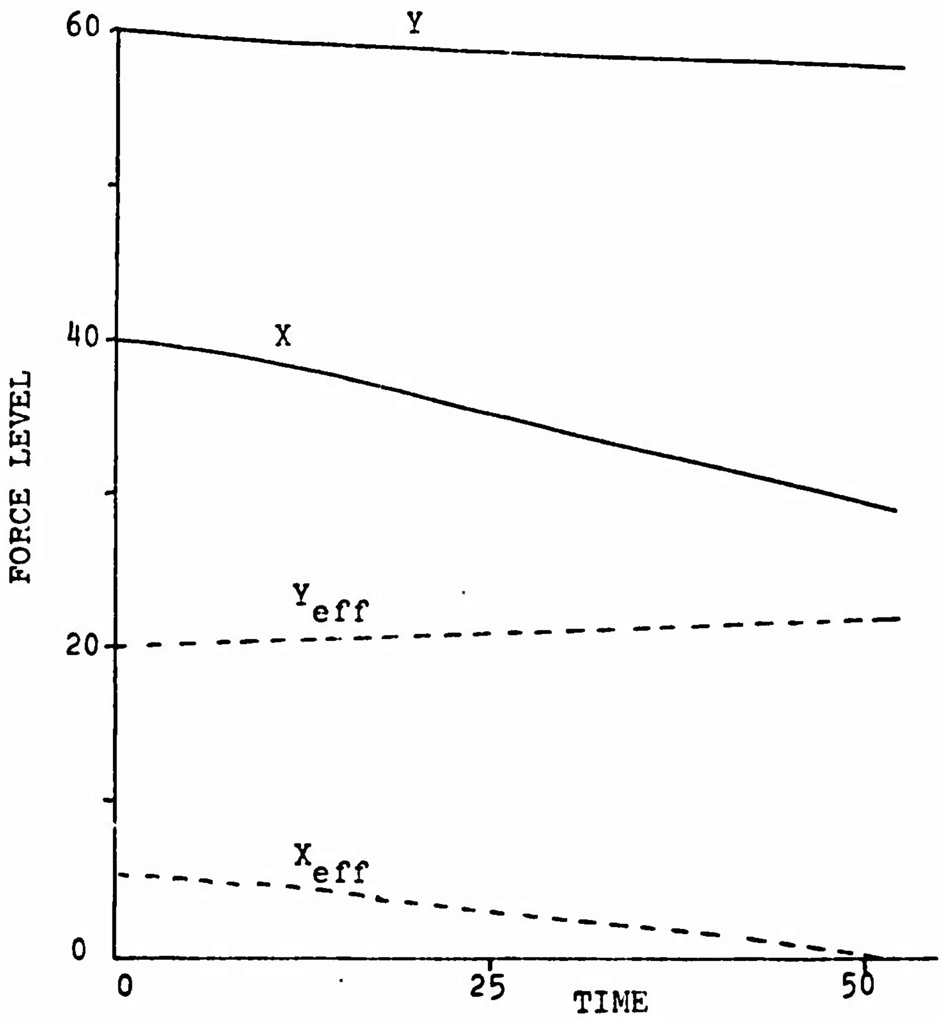

FIGURE 5B

Battle 5B Constant, Aimed, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.01	0.5	100.0	-	-
Y	60.0	0.01	0.5	100.0	-	-

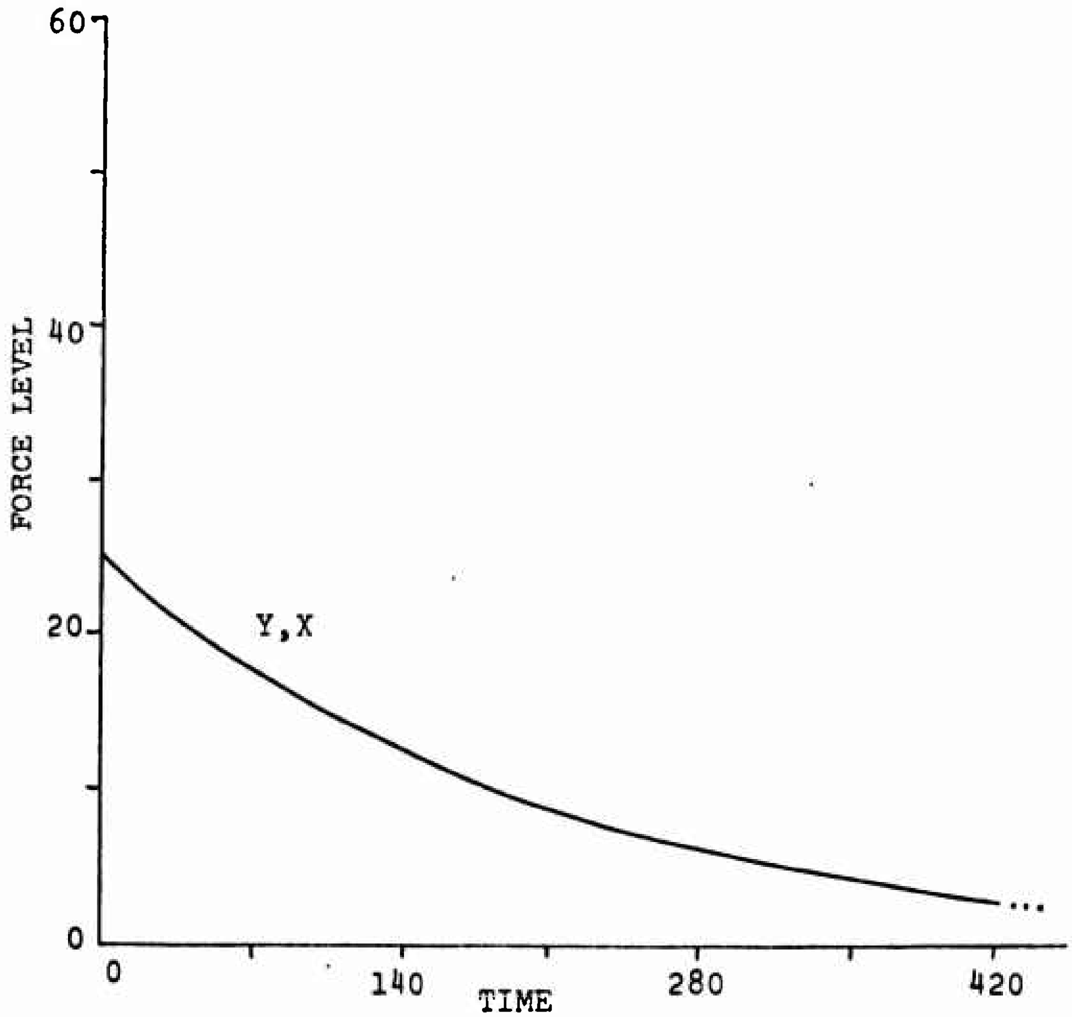

FIGURE 6A

Battle 6A Constant, Aimed, Gross Supp

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	350.0	-	-
Y	60.0	0.005	0.5	100.0	-	-

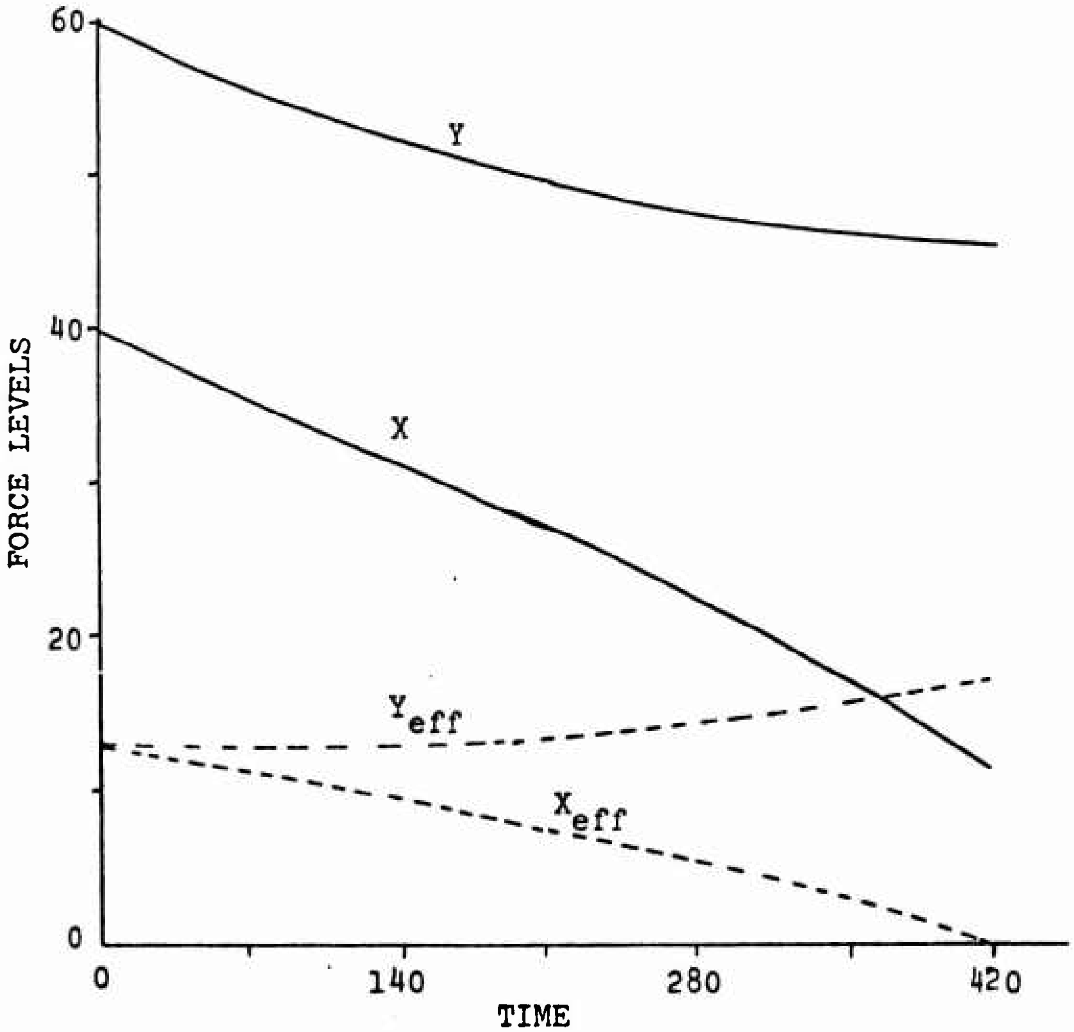

FIGURE 6B

Battle 6B Constant, Aimed, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	350.0	-	-
Y	60.0	0.005	0.5	100.0	-	-

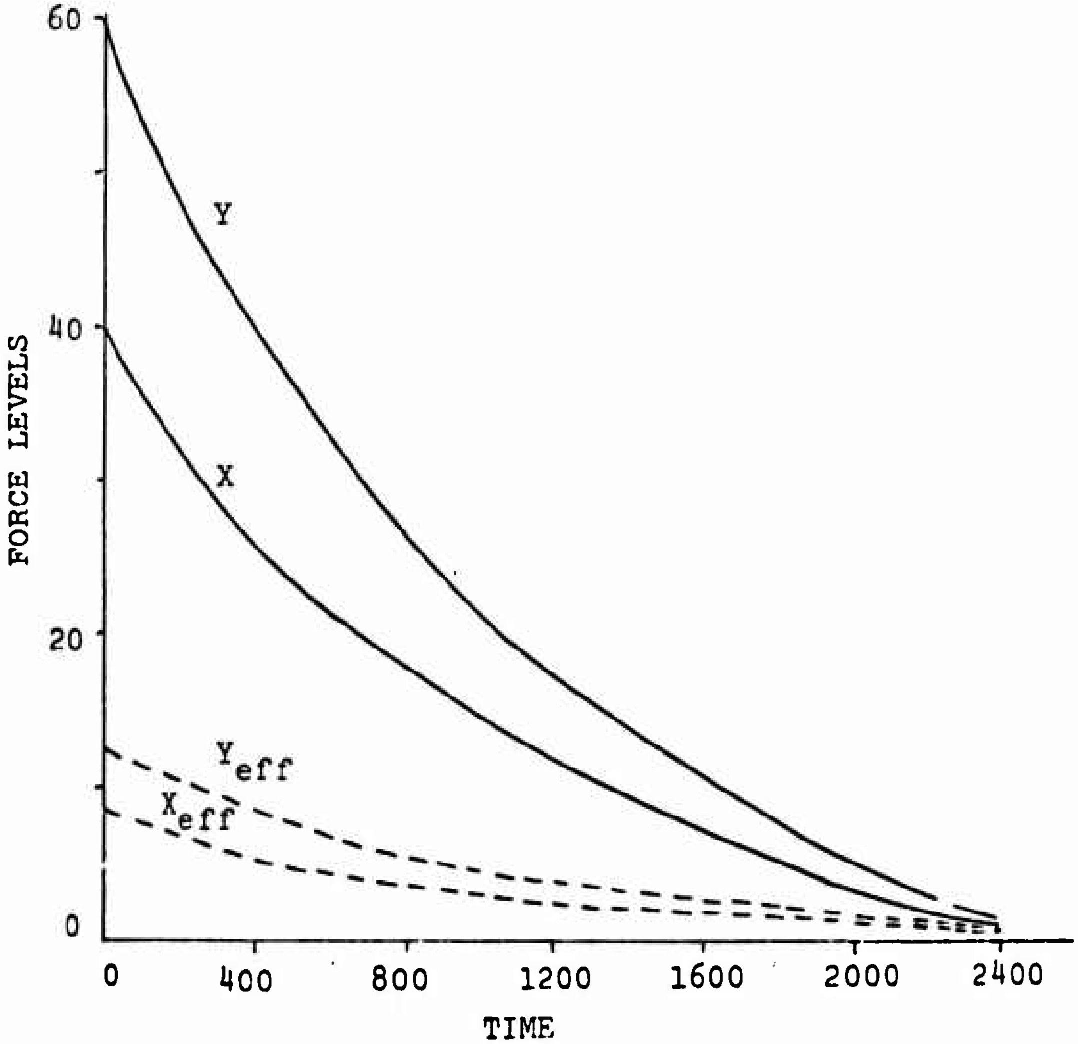

FIGURE 7

Battle 7 Constant, Aimed, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	433.3	-	-
Y	60.0	0.005	0.5	100.0	-	-

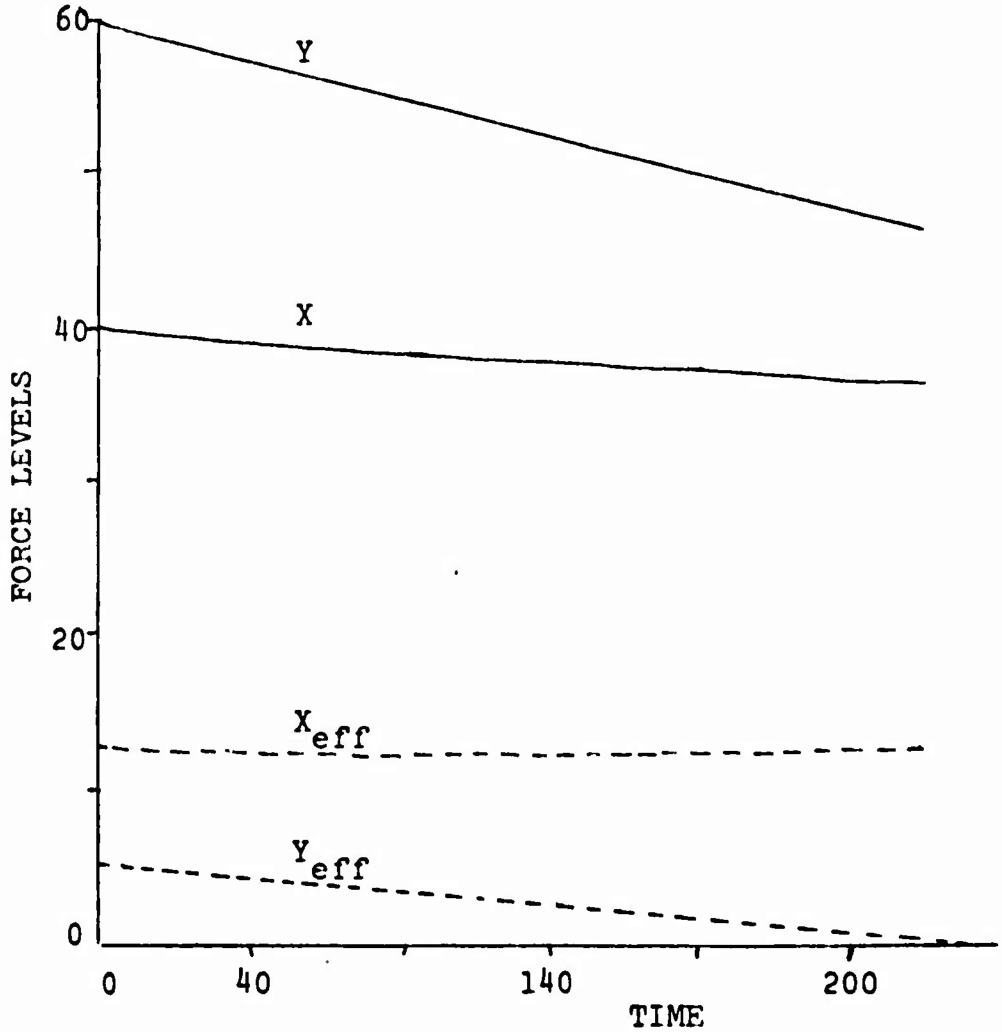

FIGURE 8

Battle 8 Constant, Aimed, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	500.0	-	-
Y	60.0	0.005	0.5	100.0	-	-

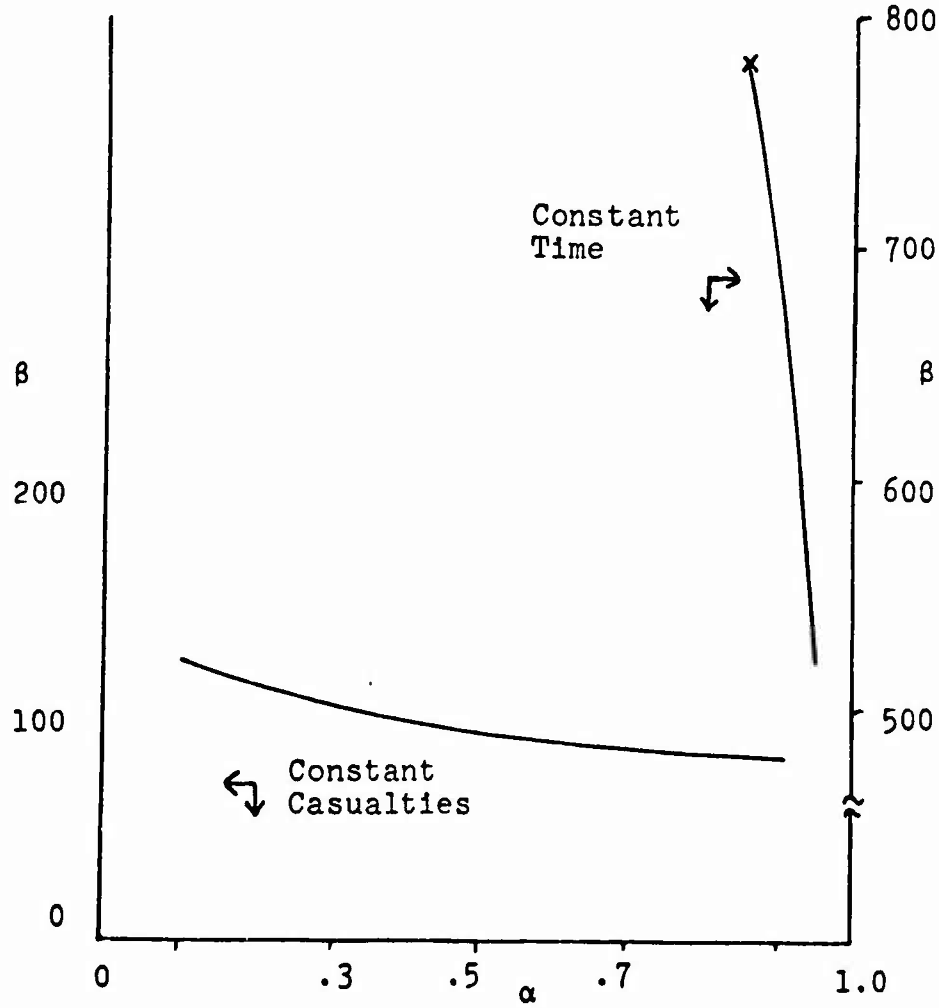

FIGURE 9A

X wins base battle in 160.9

44.7 survivors

	Base Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	60.0	0.005	1.0	-	-	-
Y	40.0	0.005	1.0	-	-	-

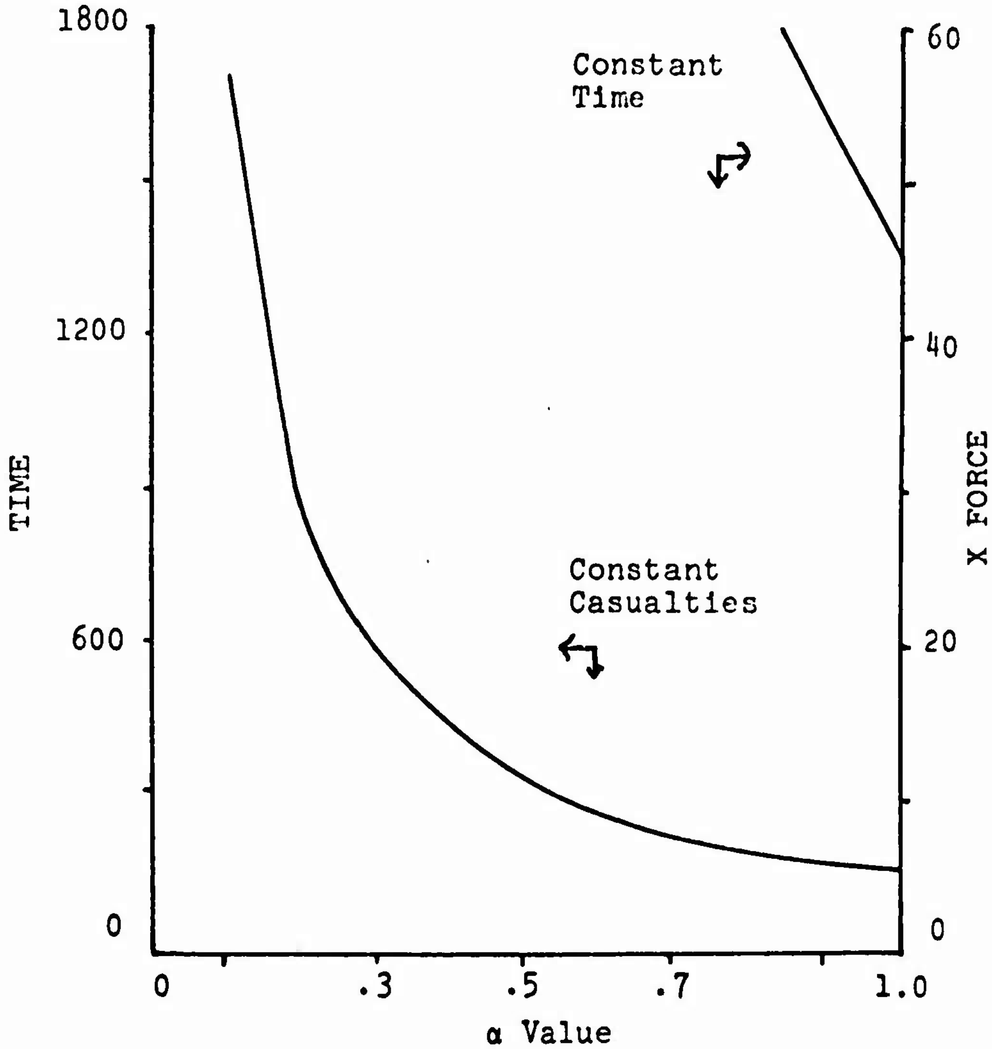

FIGURE 9B

β Chosen from curves of Figure 9A .

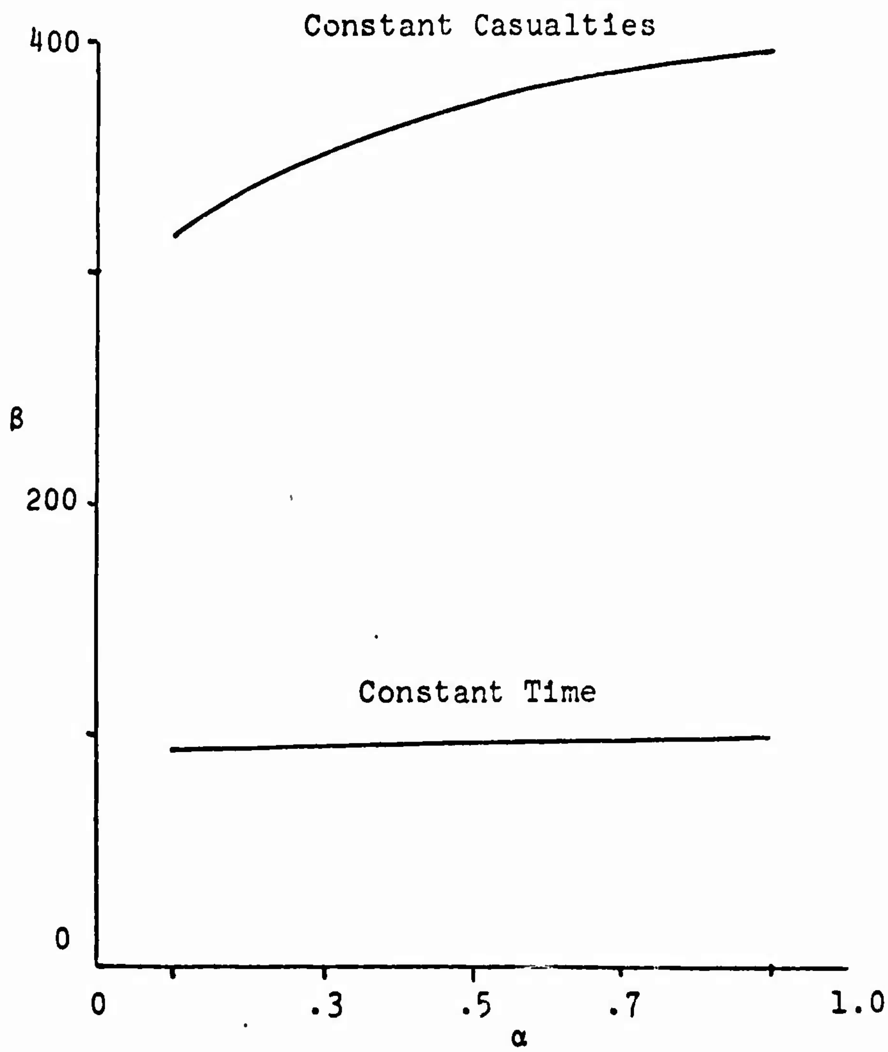

FIGURE 10A

Y wins base battle in 160.9
44.7 survivors

	Base Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	1.0	-	-	-
Y	60.0	0.005	1.0	-	-	-

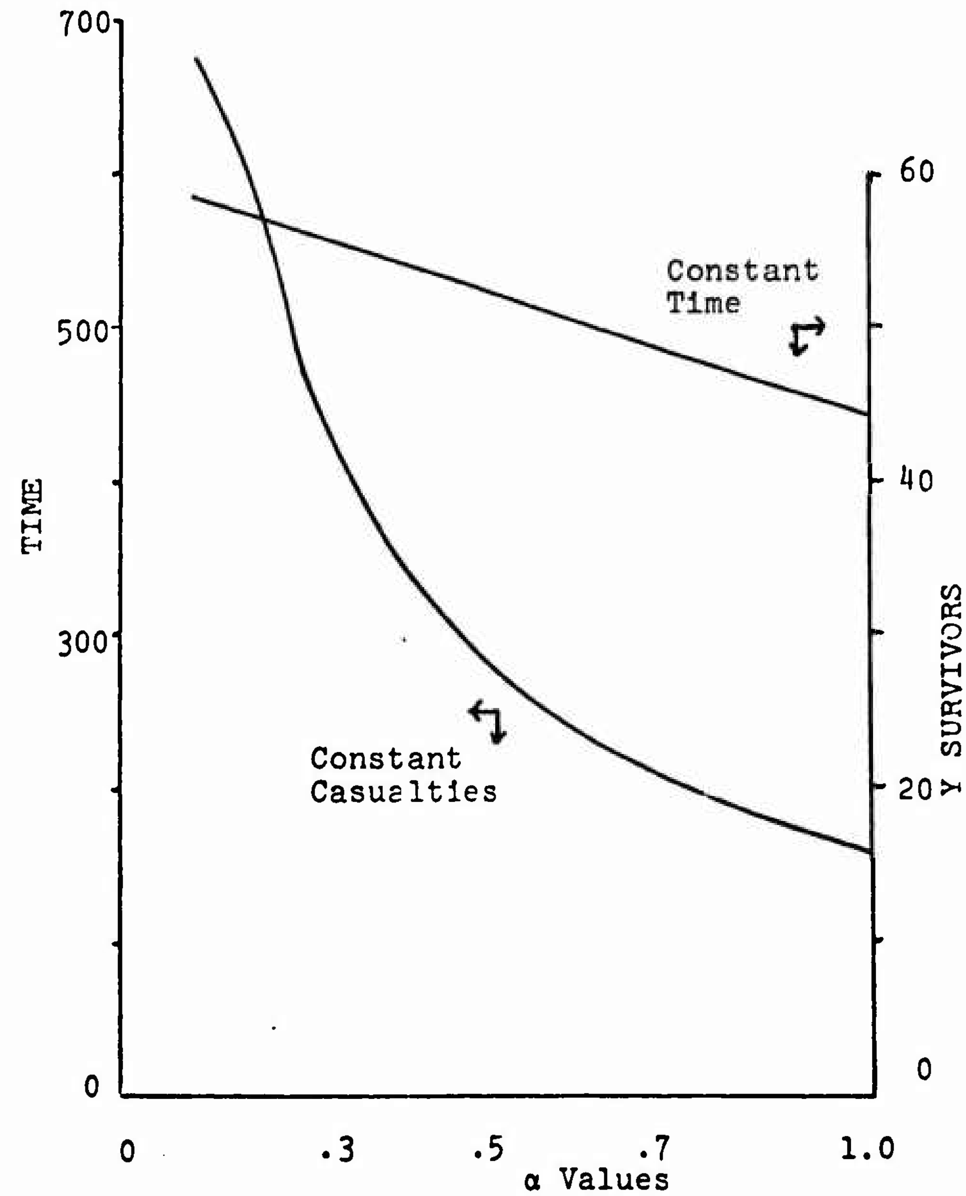

FIGURE 10B

β Chosen from curves of Figure 10A

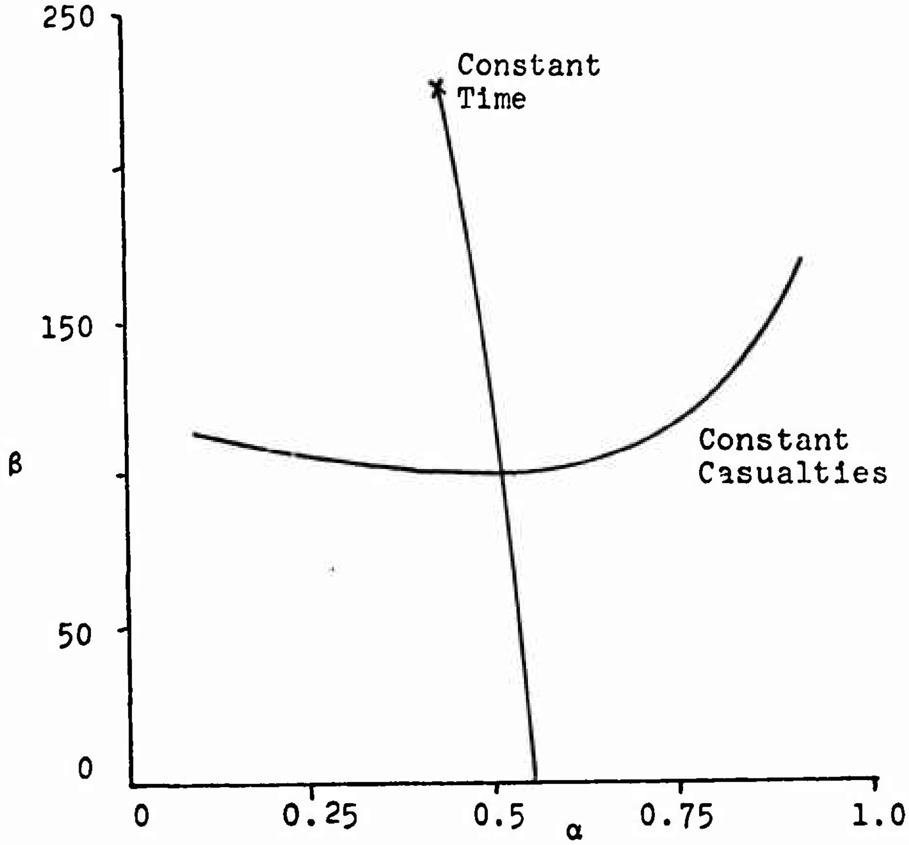

FIGURE 11A

X wins base battle in 319.7
52.99 survivors

	Base Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	60.0	0.005	0.5	150.0	-	-
Y	40.0	0.005	0.5	100.0	-	-

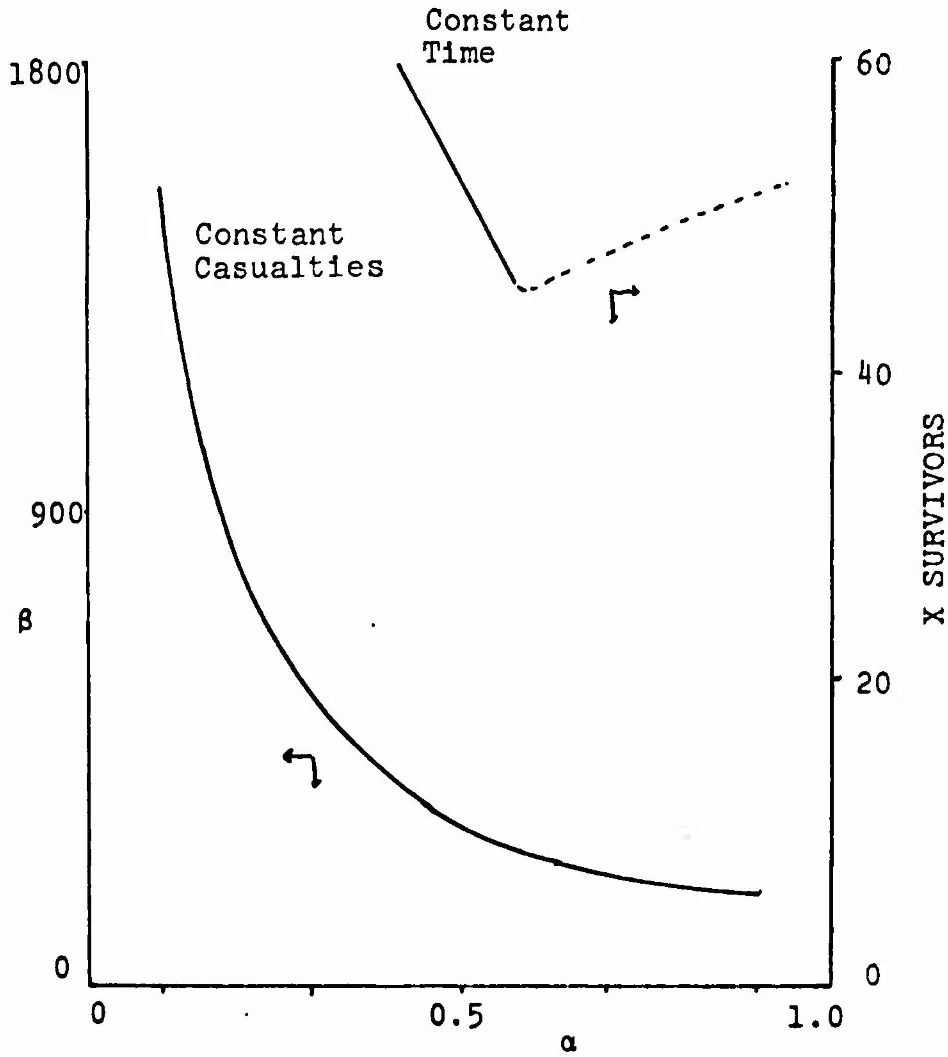

FIGURE 11B

β chosen from curves of Figure 11A

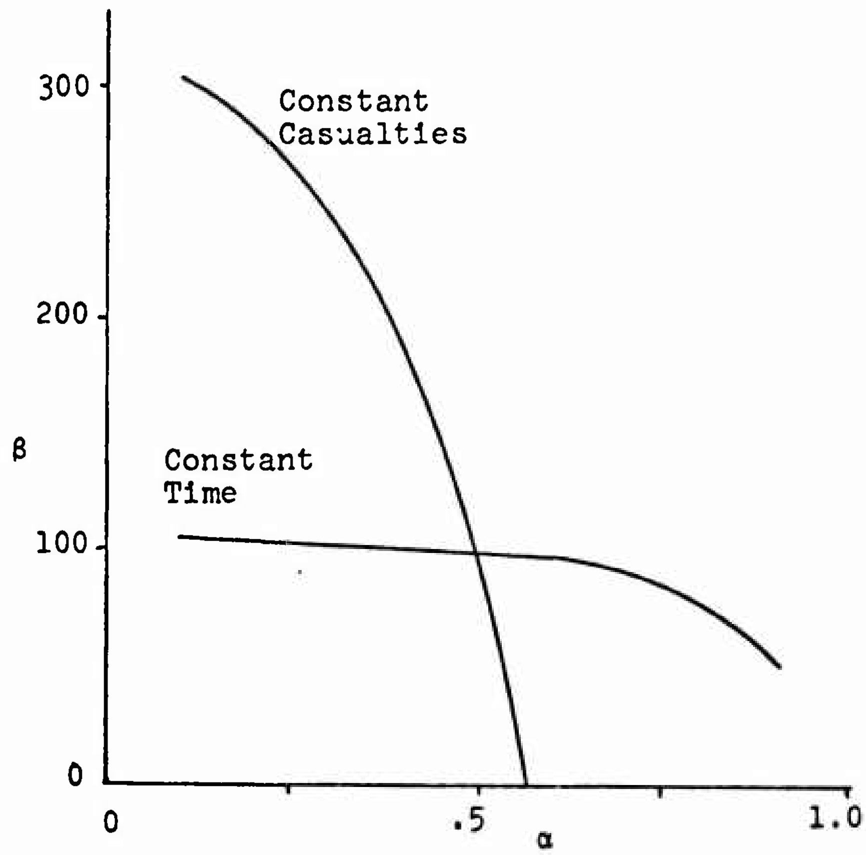

FIGURE 12A

Y wins base battle in 319.7
52.9 survivors

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	100.0	-	-
Y	60.0	0.005	0.5	100.0	-	-

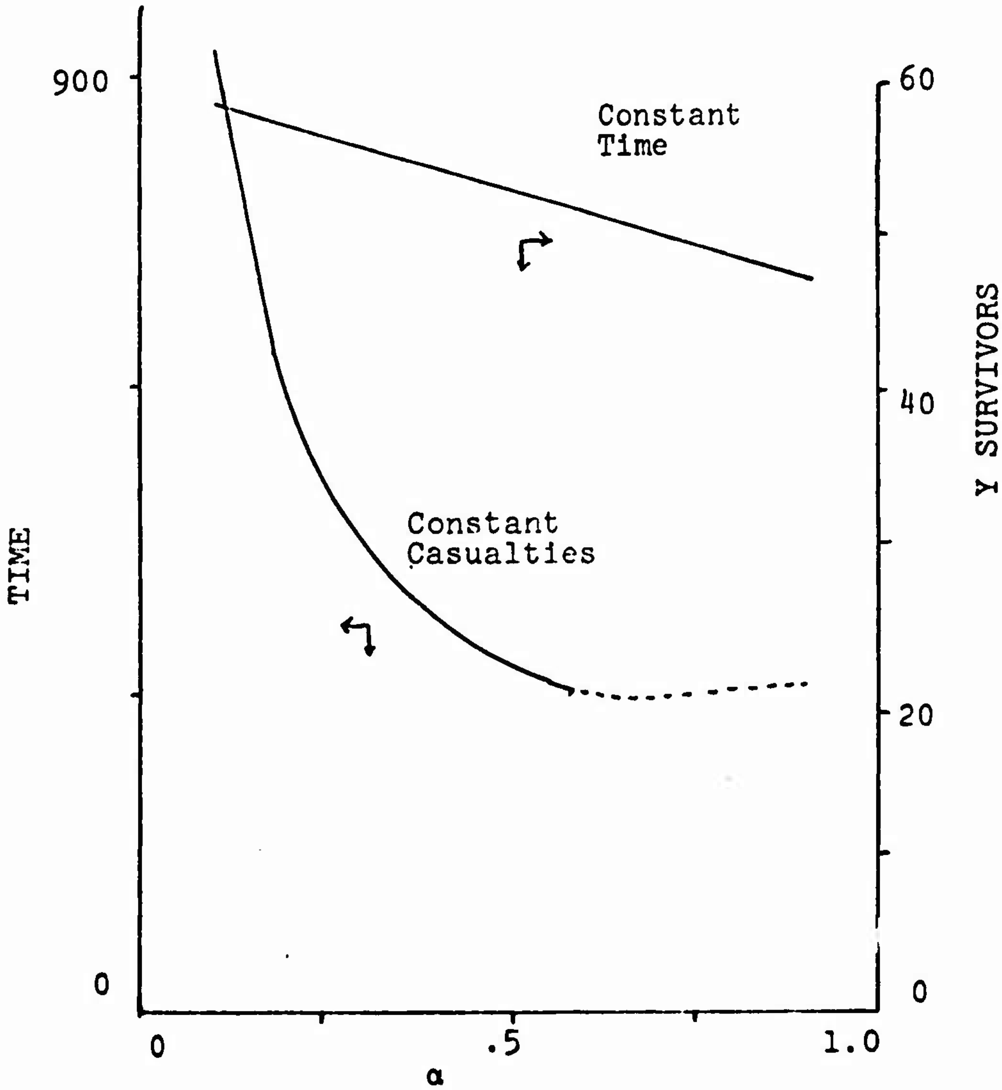

FIGURE 12B

β Values from Figure 12A

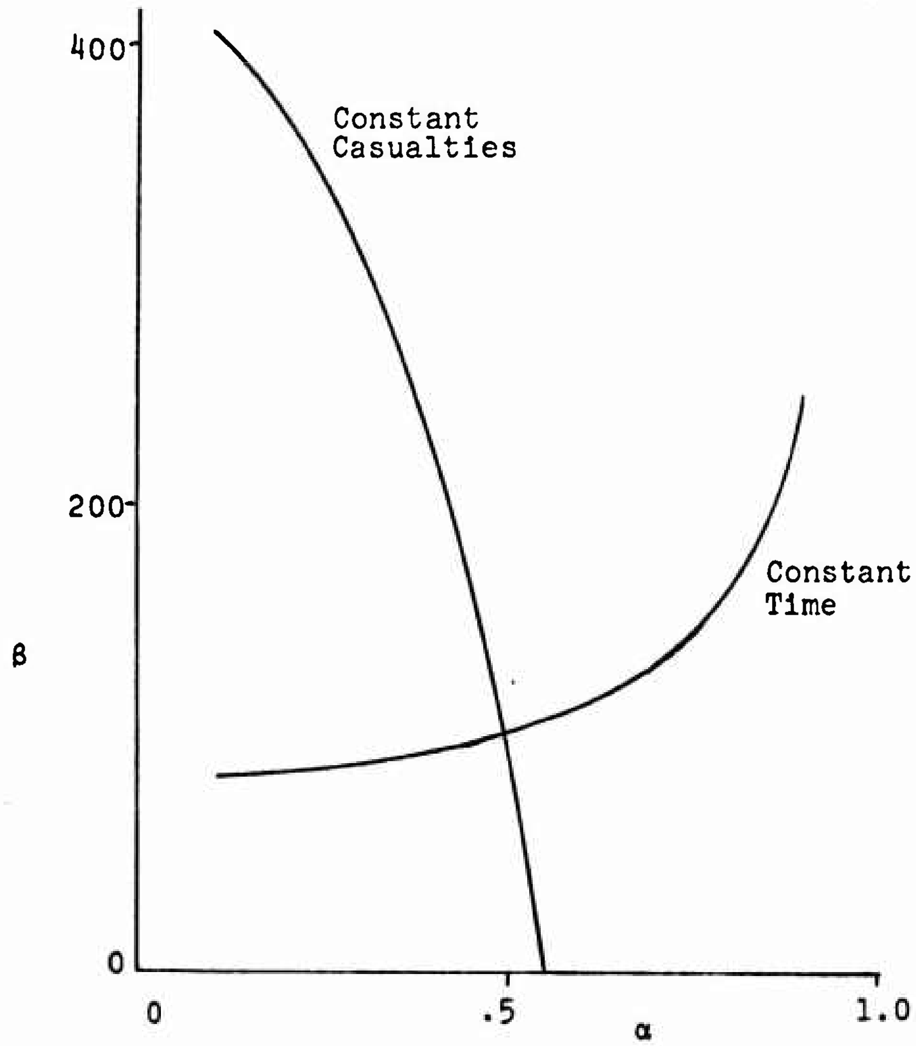

FIGURE 13A

Y wins base battle in 217.6
77.01 survivors

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	100.0	-	-
Y	80.0	0.005	0.5	100.0	-	-

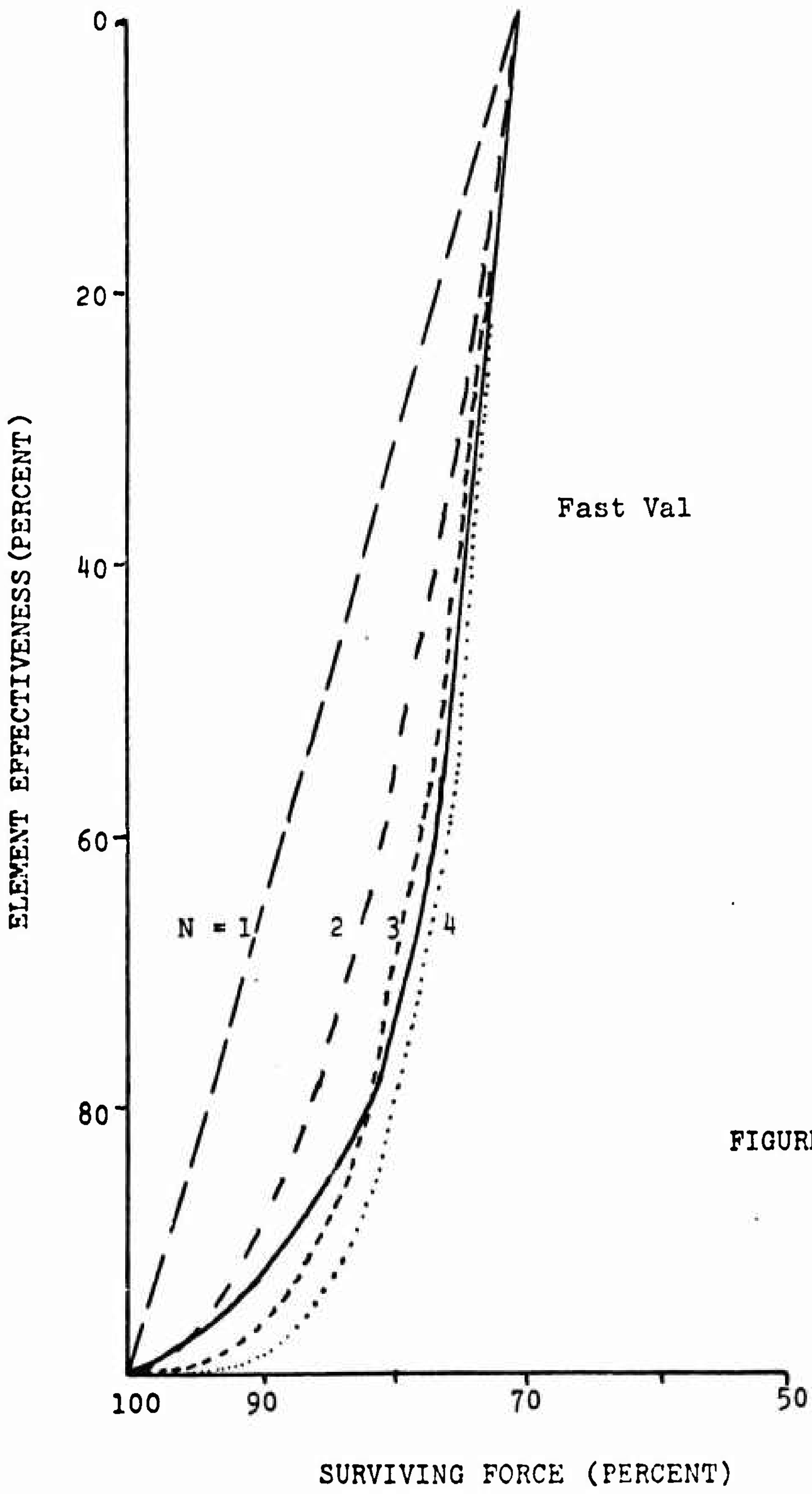

FIGURE 14

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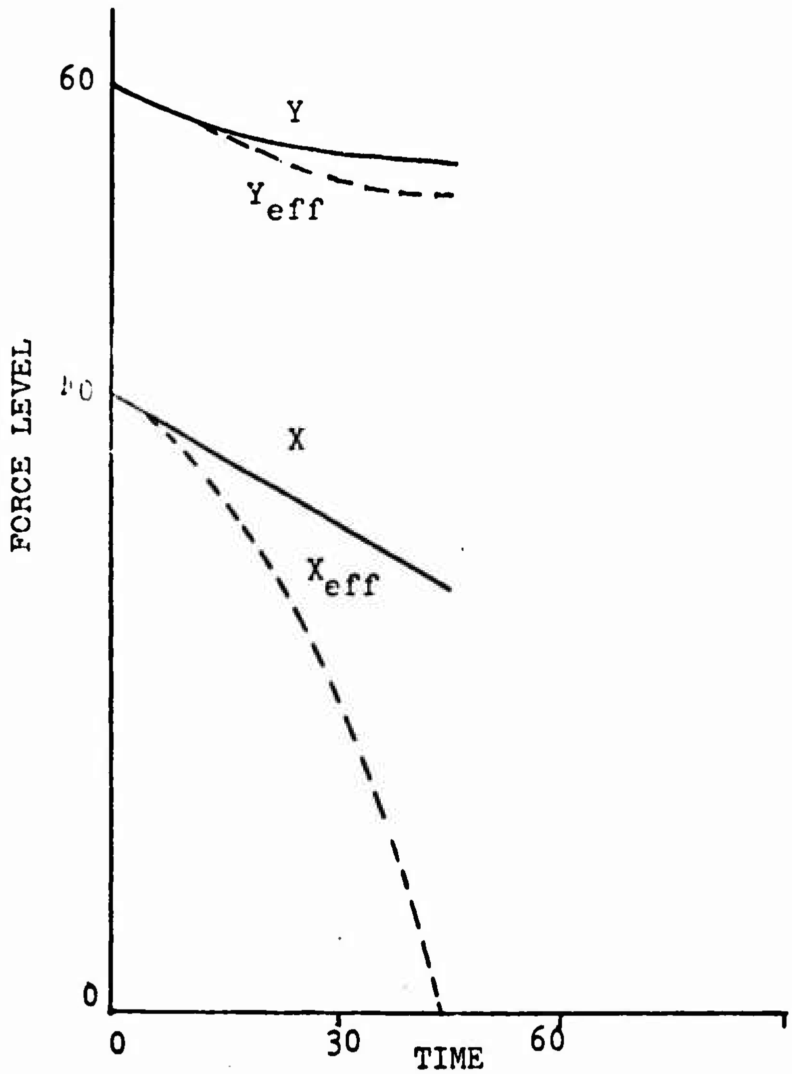

FIGURE 15

Battle 9 Nonconstant, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	-	-	0.7	3.0
Y	60.0	0.005	-	-	0.7	3.0

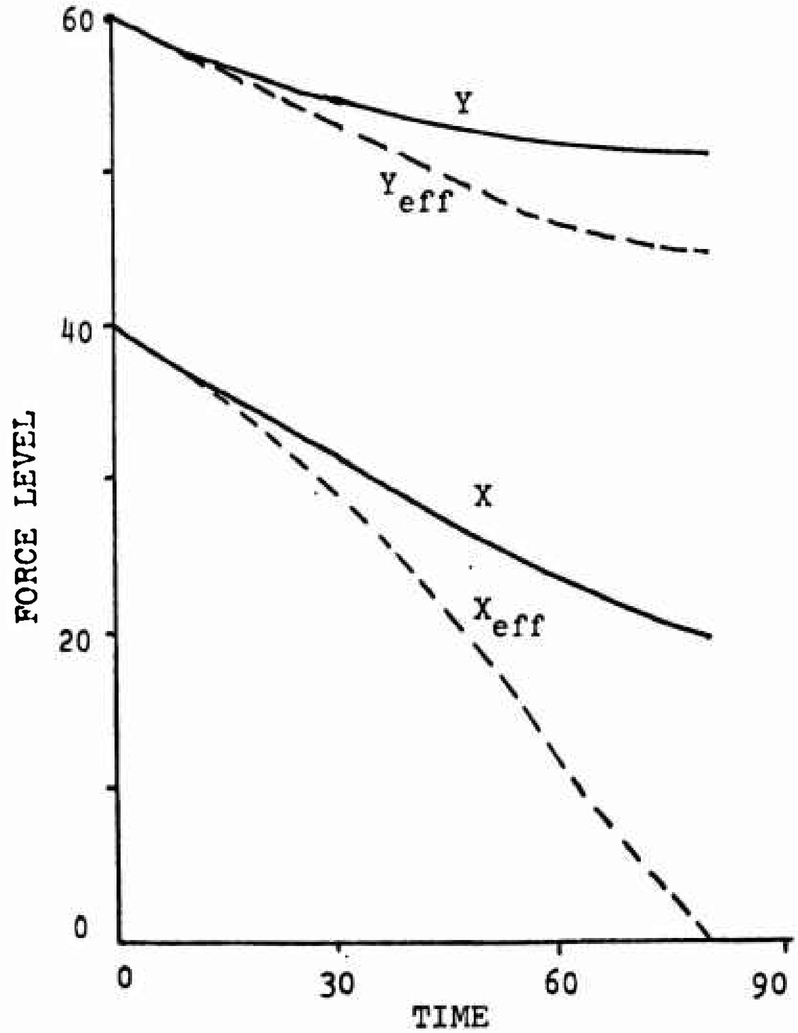

FIGURE 16

Battle 10 Nonconstant, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	-	-	0.5	3.0
Y	60.0	0.005	-	-	0.7	3.0

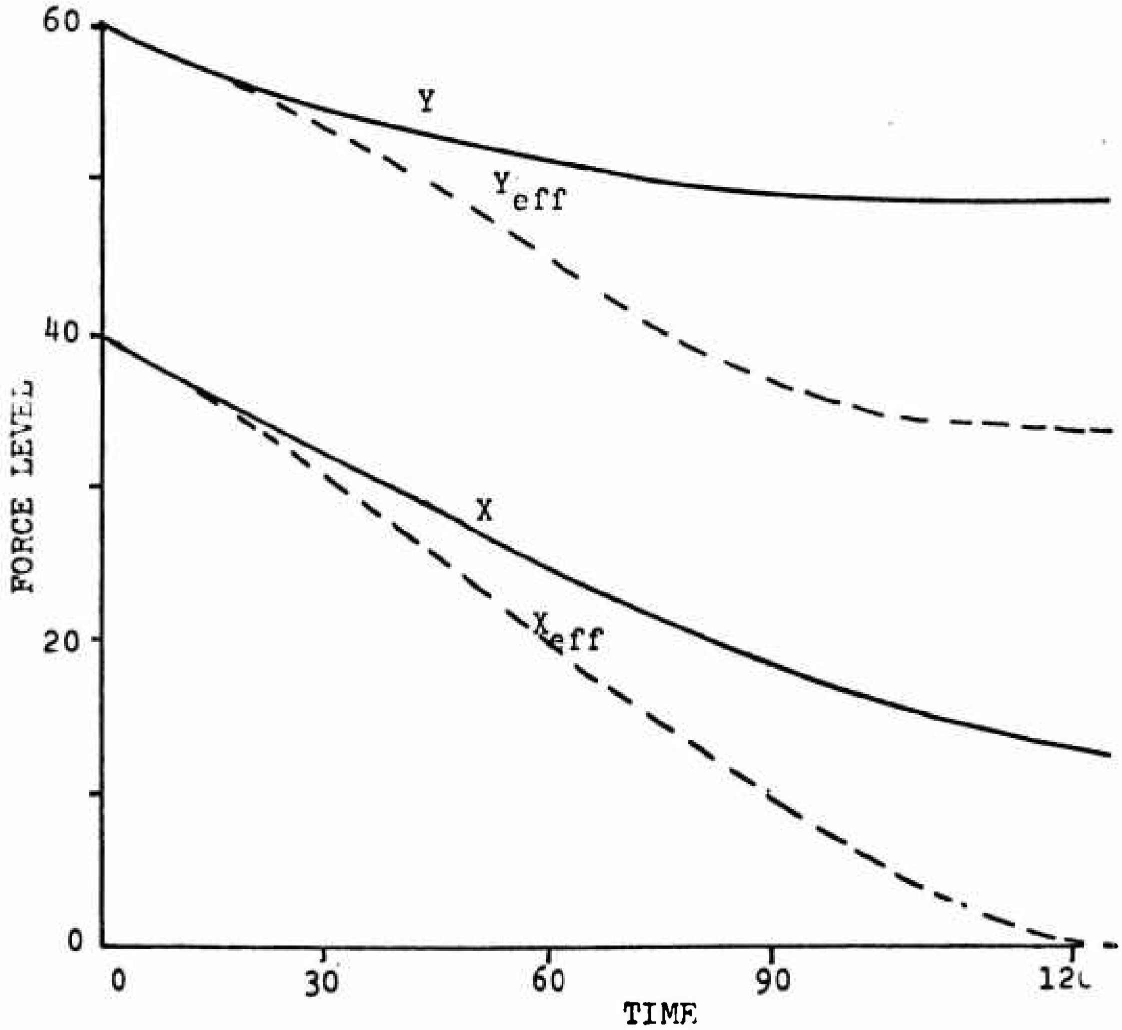

FIGURE 17

Battle 11 Nonconstant, aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	-	-	0.3	3.0
Y	60.0	0.005	-	-	0.7	3.0

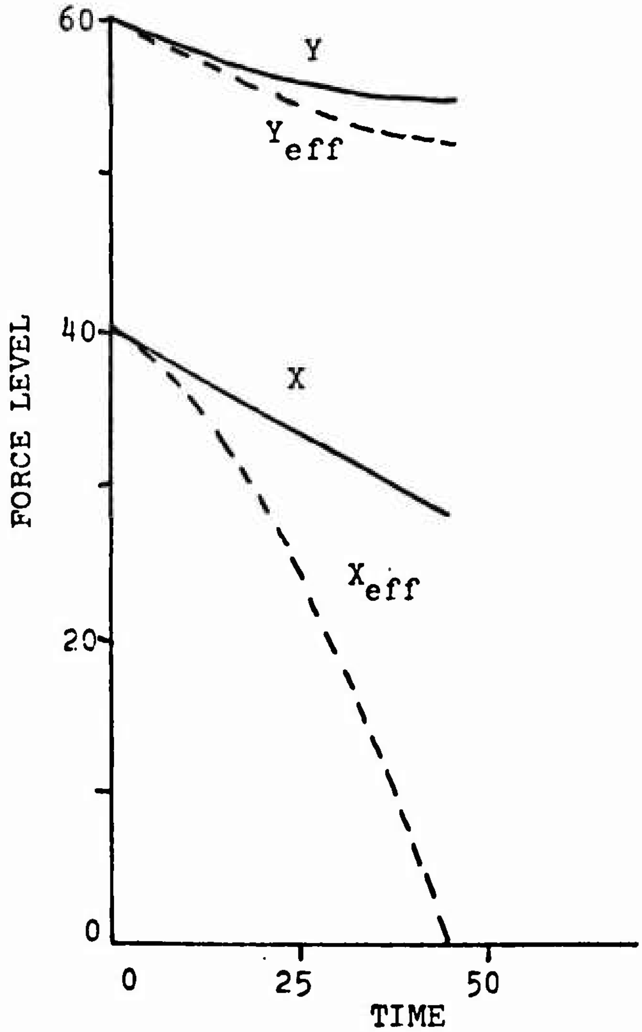

FIGURE 18

Battle 12 Nonconstant, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	-	-	0.7	2.5
Y	60.0	0.005	-	-	0.7	2.5

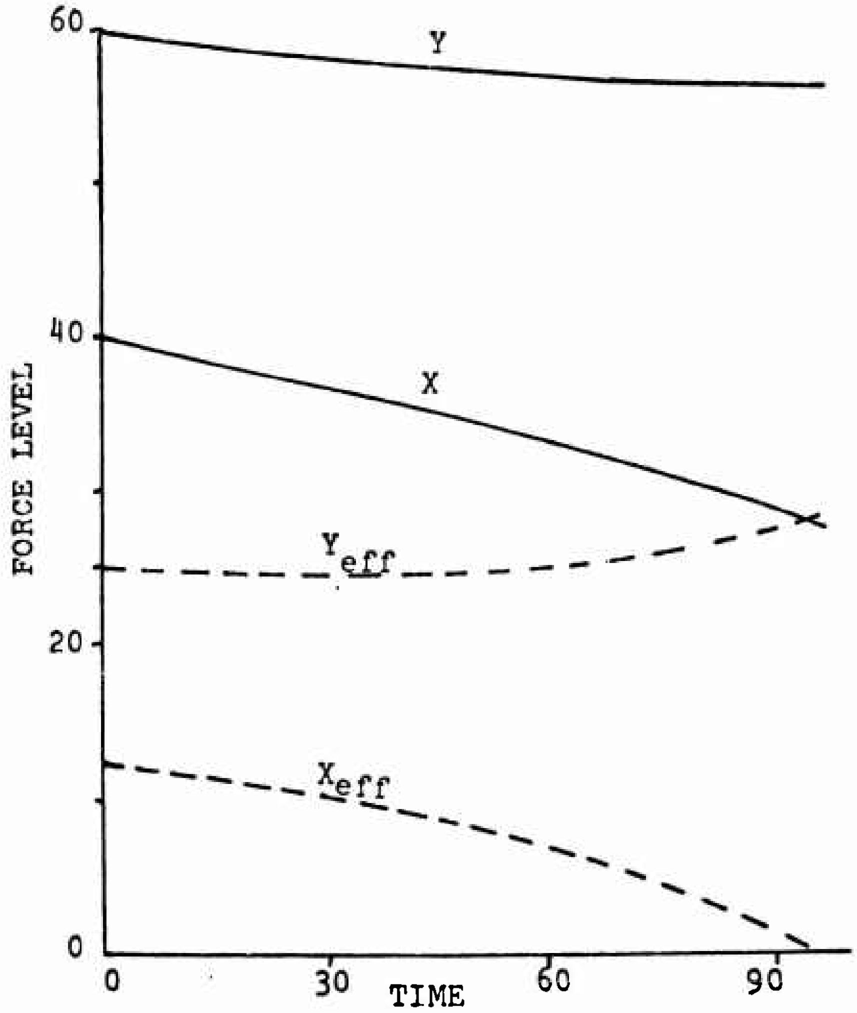

FIGURE 19

Battle 13 Nonconstant, Aimed, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	100.0	0.7	3.0
Y	60.0	0.005	0.5	100.0	0.7	3.0

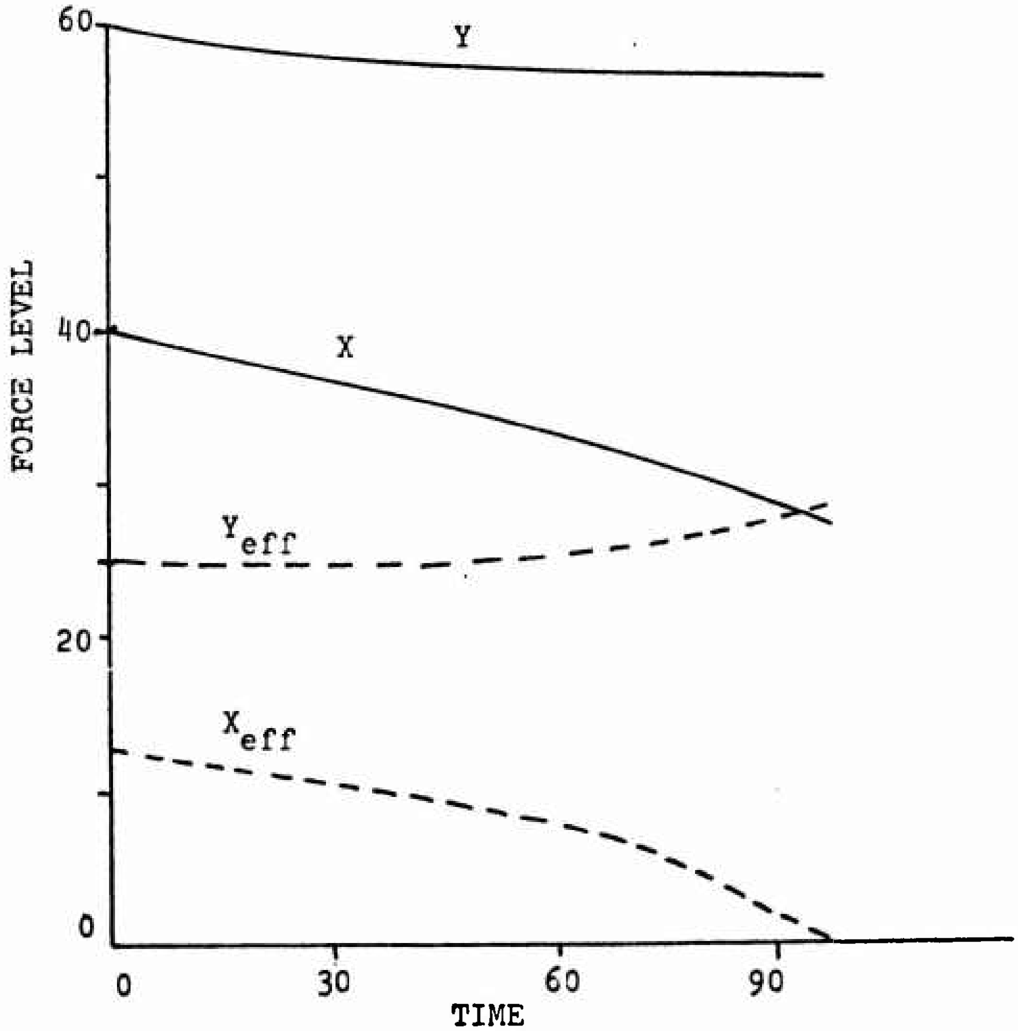

FIGURE 20

Battle 14 Nonconstant, Aimed, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	100.0	0.7	3.5
Y	60.0	0.005	0.5	100.0	0.7	3.5

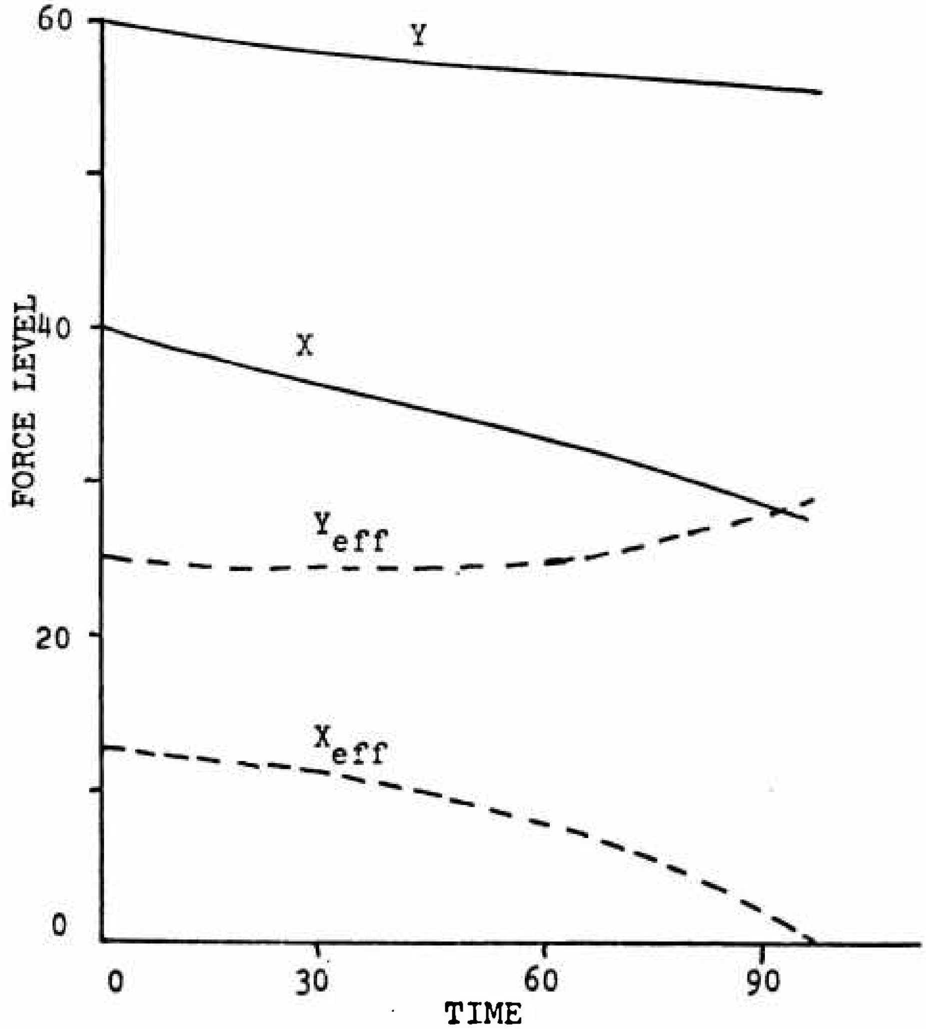

FIGURE 21

Battle 15 Nonconstant, Aimed, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	100.0	0.7	4.0
Y	60.0	0.005	0.5	100.0	0.7	4.0

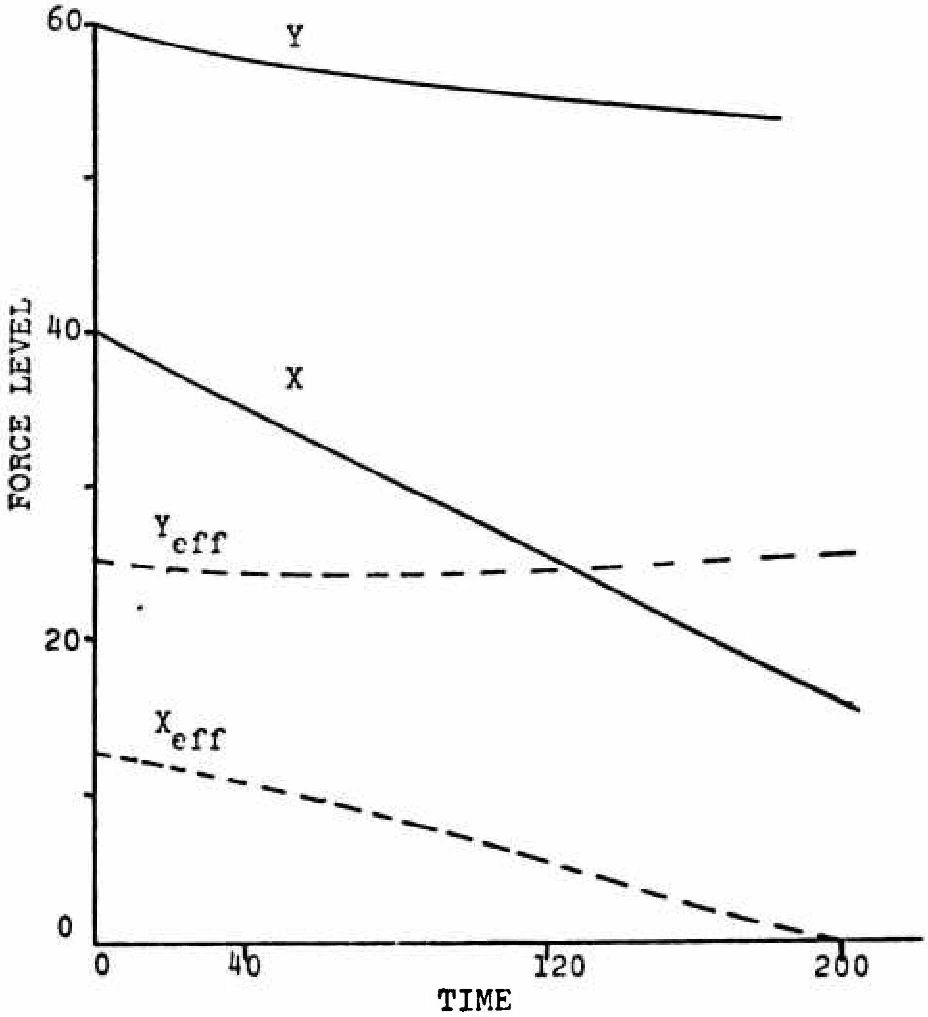

FIGURE 22

Battle 16 Nonconstant, Aimed, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	100.0	0.3	3.0
Y	60.0	0.005	0.5	100.0	0.7	3.0

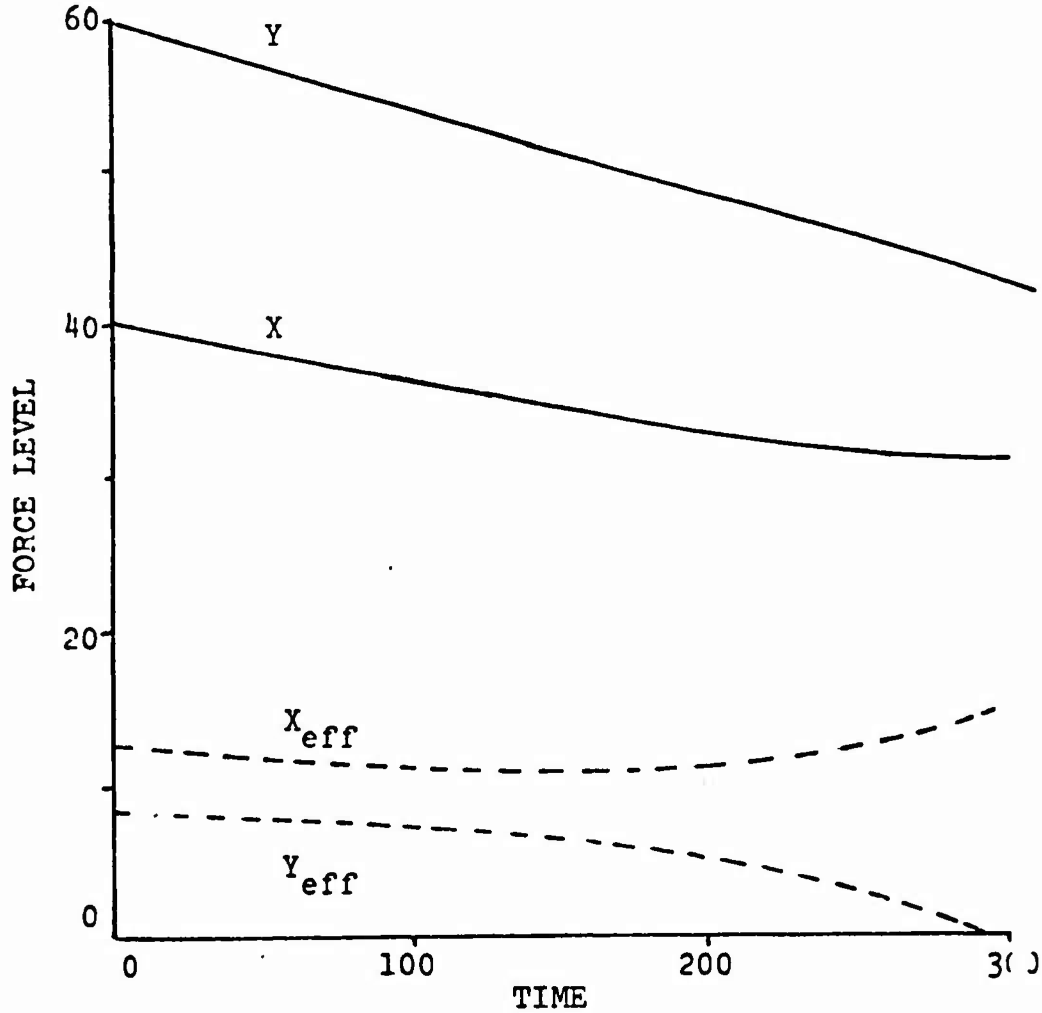

FIGURE 23

Battle 17 Nonconstant, Aimed, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	433.3	0.5	3.0
Y	60.0	0.005	0.5	100.0	0.7	3.0

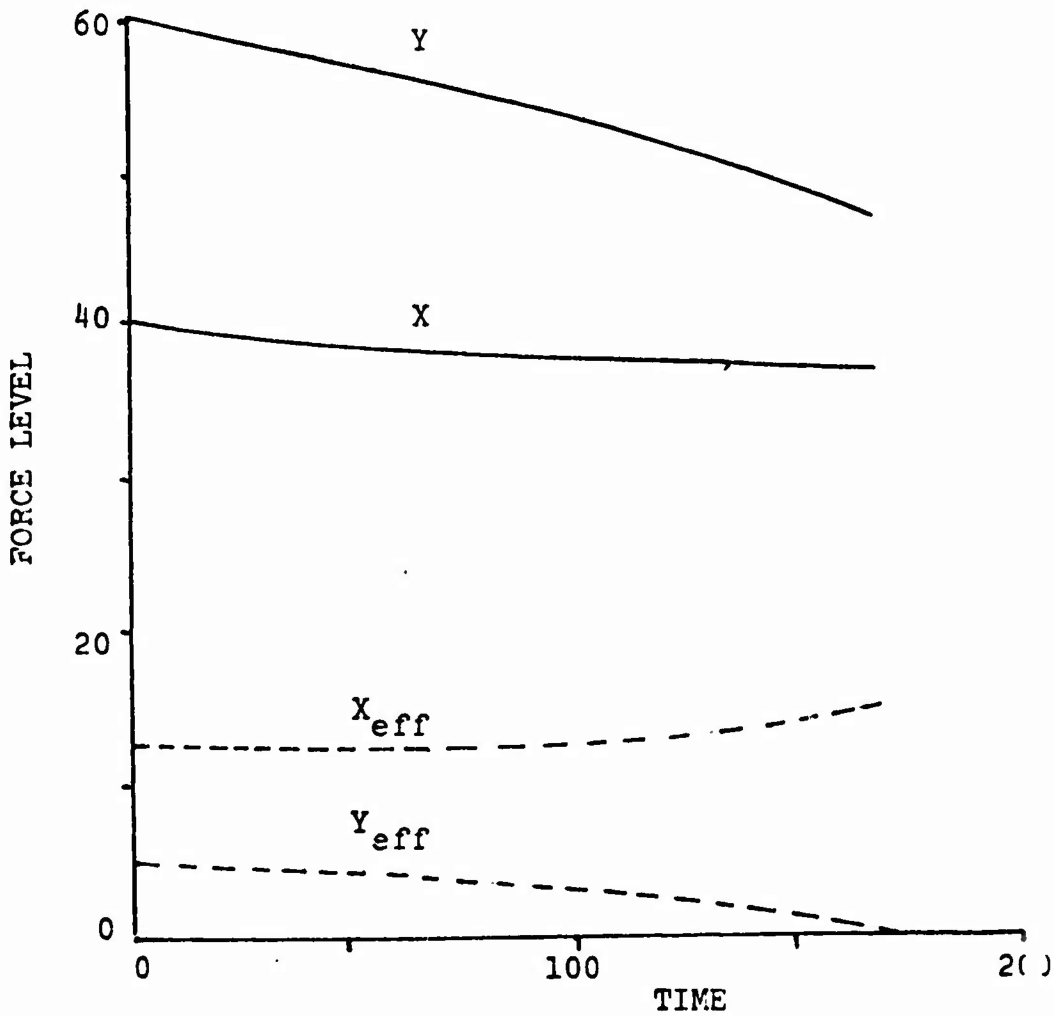

FIGURE 24

Battle 18 Nonconstant, Aimed, Aimed

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	500.0	0.7	3.0
Y	60.0	0.005	0.5	100.0	0.7	3.0

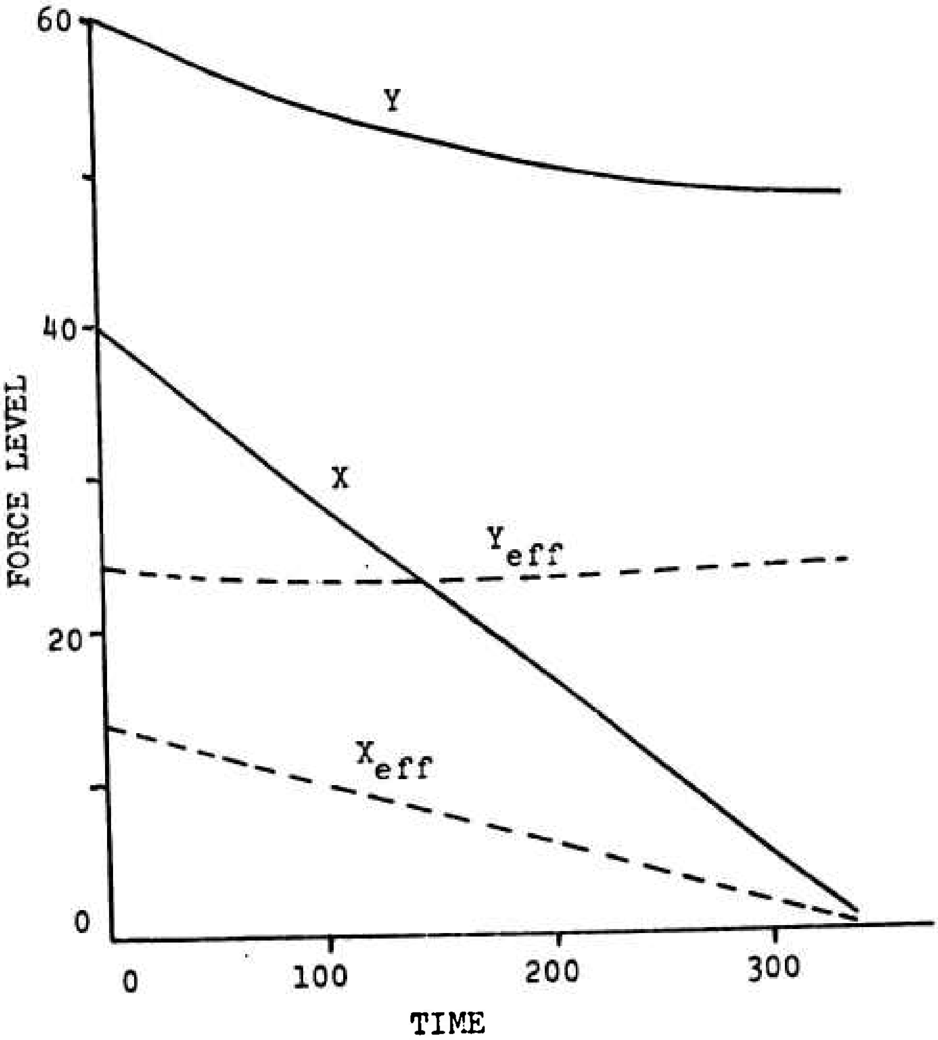

FIGURE 25

Battle 19 Constant, Aimed, Area

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	2.0	-	
Y	60.0	0.005	0.5	2.0		

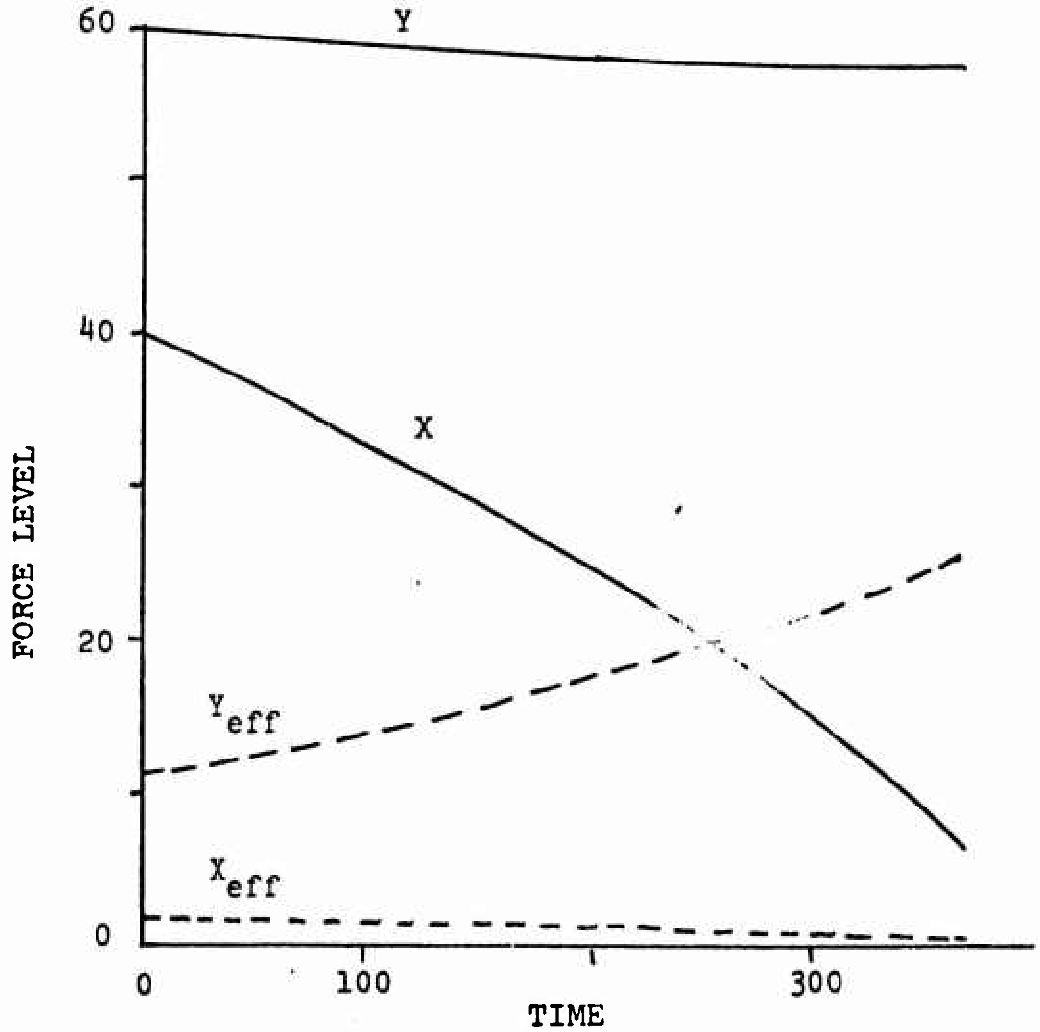

FIGURE 26

Battle 20 Constant, Aimed area

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	6.0	-	-
Y	60.0	0.005	0.5	6.0	-	-

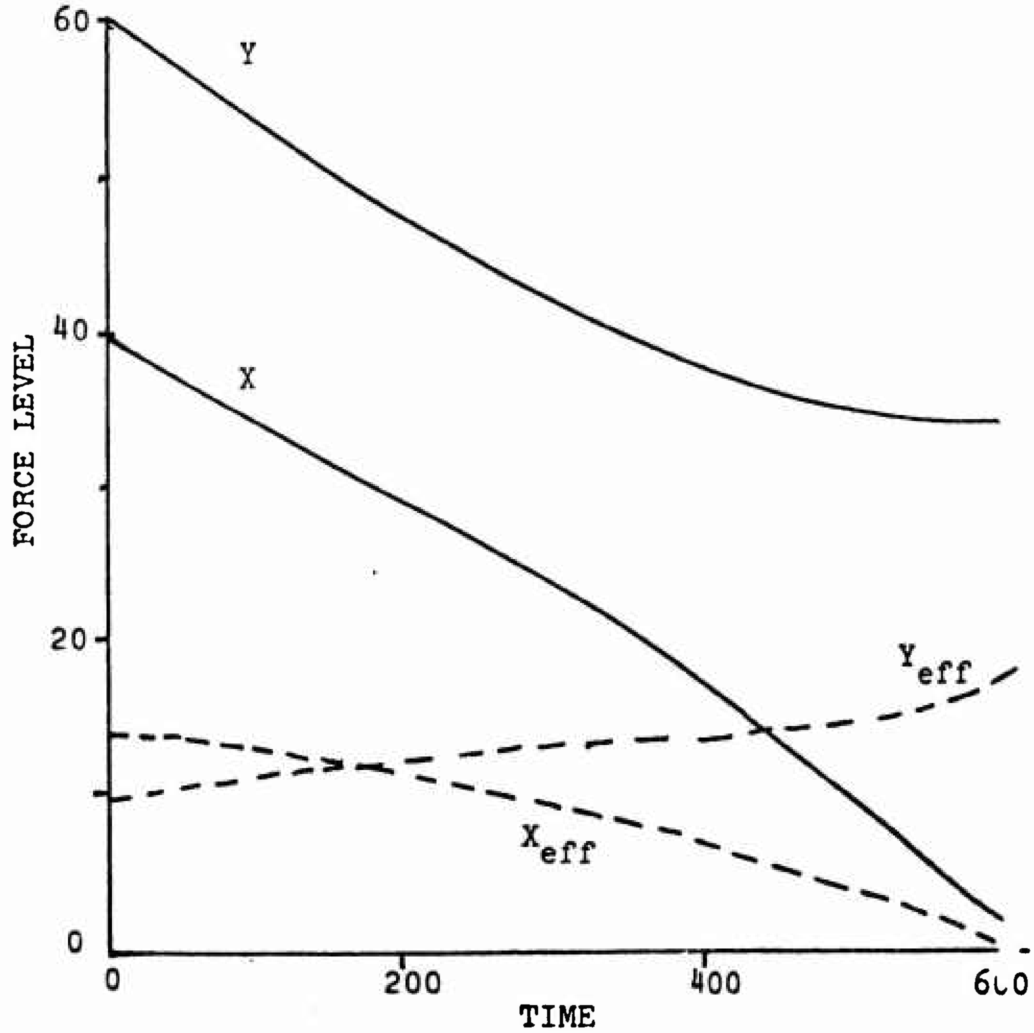

FIGURE 27

Battle 21 Constant, Aimed, Area

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	6.89	-	-
Y	60.0	0.005	0.5	2.00	-	-

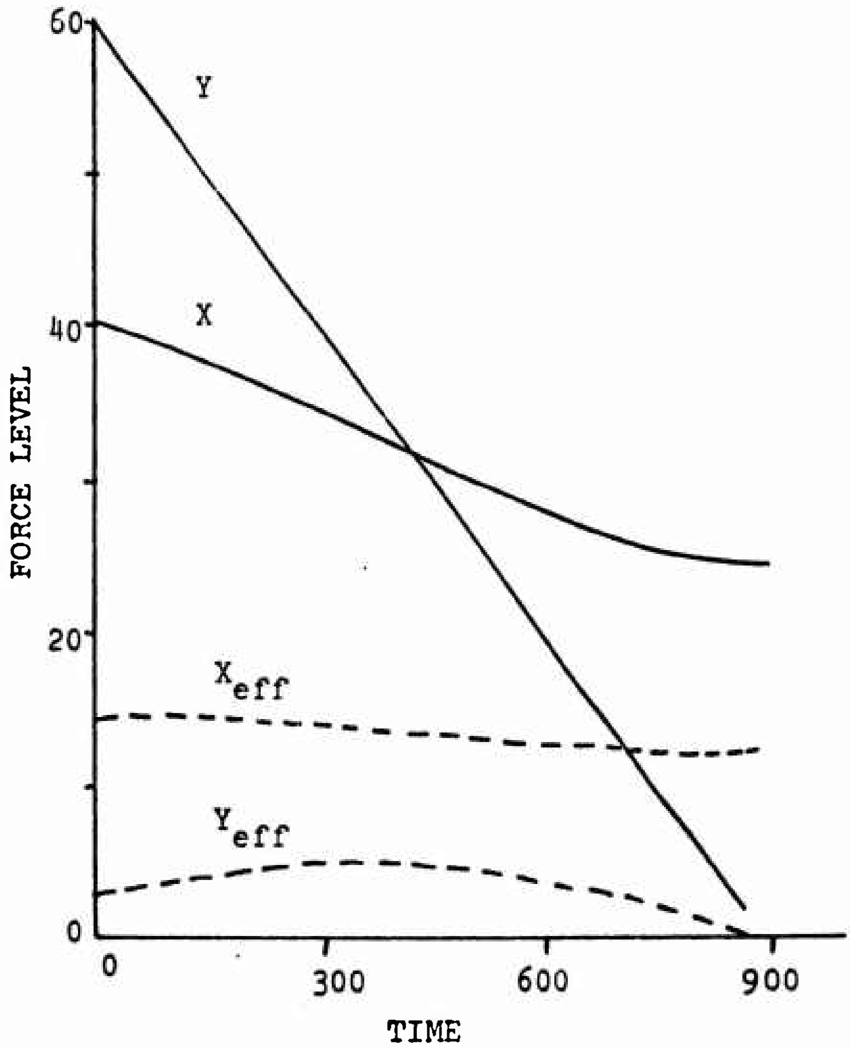

FIGURE 28

Battle 22 Constant, Aimed, Area

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.005	0.5	9.0	-	-
Y	60.0	0.005	0.5	2.0	-	-

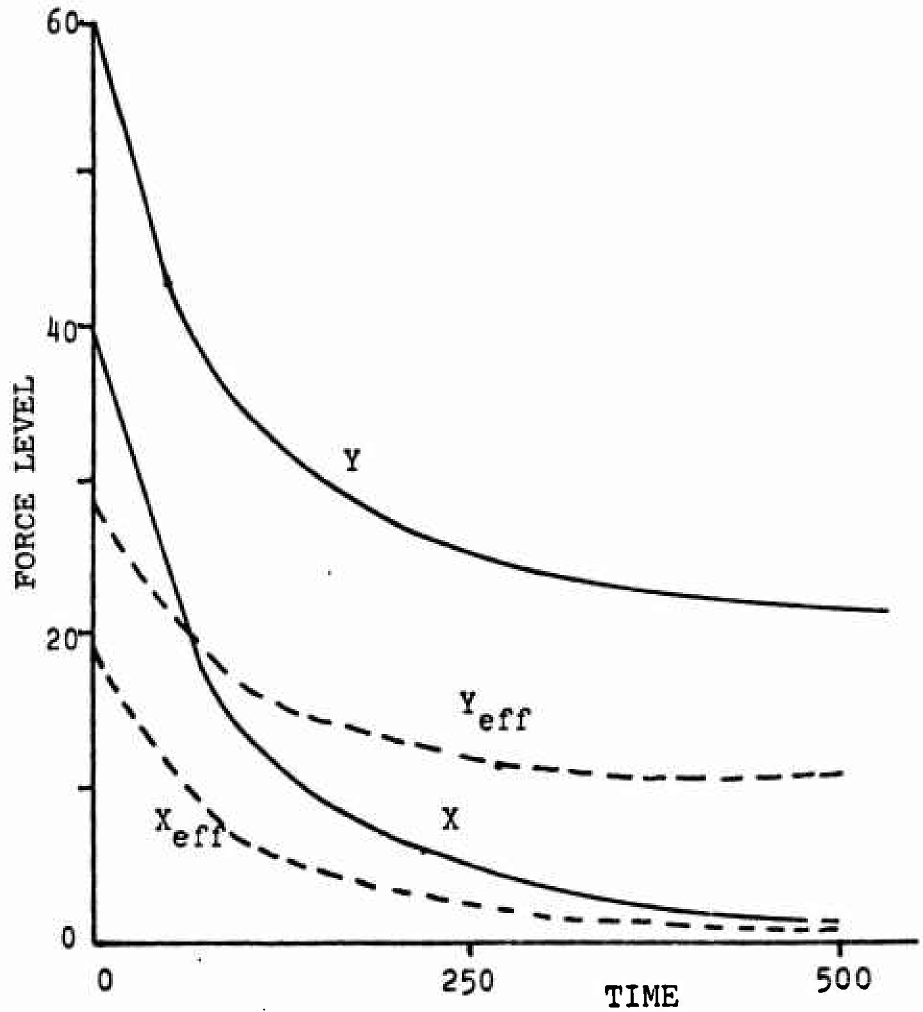

FIGURE 29

Battle 23 Constant, Area, Area

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.0005	0.5	2.0	-	-
Y	60.0	0.0005	0.5	2.0	-	-

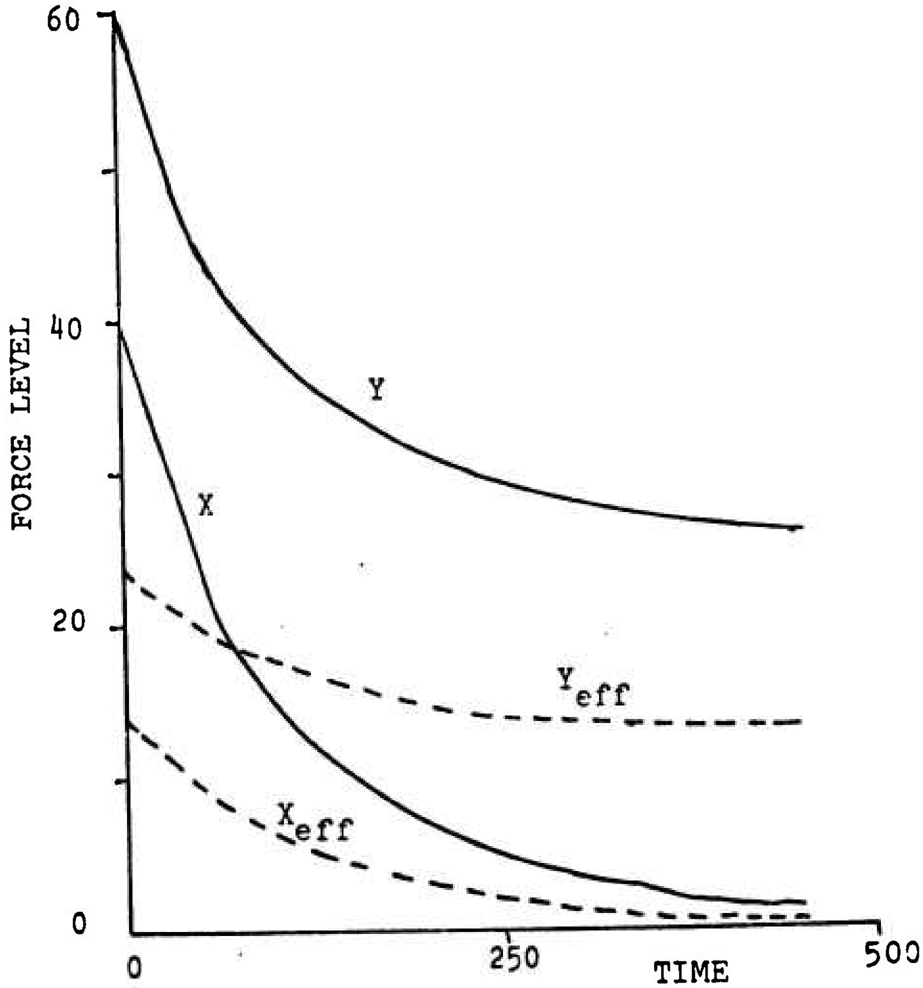

FIGURE 30

Battle 24 Constant, Area, Area

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.0005	0.5	20.0	-	-
Y	60.0	0.0005	0.5	20.0	-	-

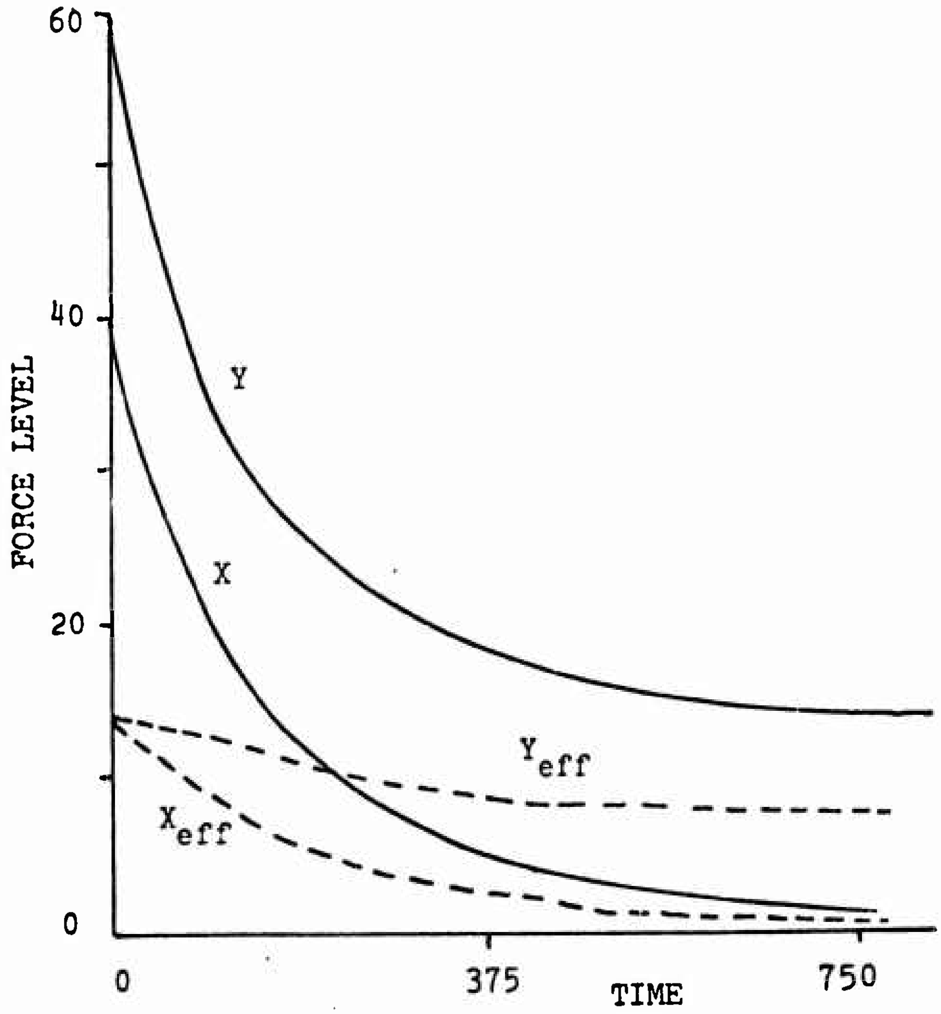

FIGURE 31

Battle 25 Constant, Area, Area

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.0005	0.5	53.33	-	-
Y	60.0	0.0005	0.5	20.00	-	-

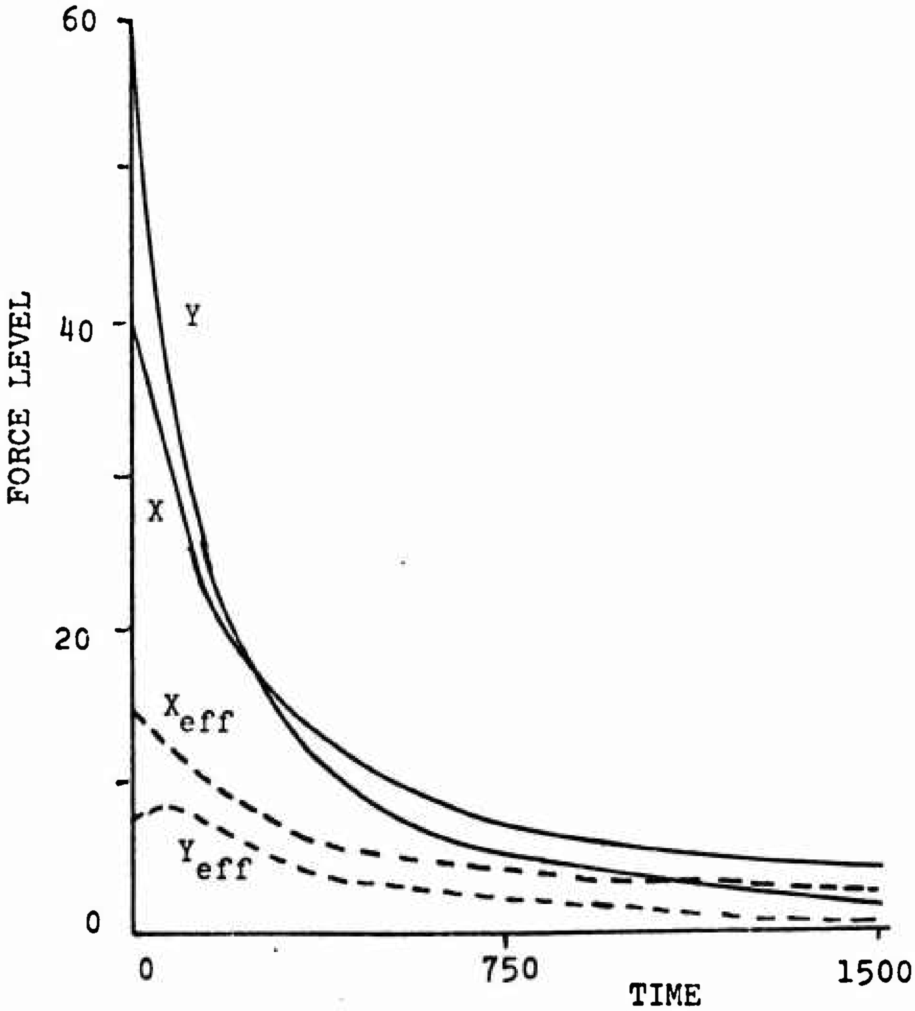

FIGURE 32

Battle 26 Constant, Area, Area

	Force	Atr Coef	Frac Cas	Supp Eff	Bk Pt	Shape
X	40.0	0.0005	0.5	75.0	-	-
Y	60.0	0.0005	0.5	20.0	-	-

COMPUTER PROGRAM

```
      FOR VALUES OF ALFA AT INTERVALS OF SIZE STEP PROGRAM COMPUTES THE
BETA VALUE REQUIRED TO GIVE EITHER A FIXED COMBAT TIME OR FIXED LOSSES
FOR THE WINNER OF THE BASE BATTLE.
THE PROGRAM DETERMINES THE WINNER OF THE BASE BATTLE FOR FORCE PARAMETERS
X = THE INITIAL STRENGTH OF SIDE X WITH ATTRITION COEFFICIENT B
LETHALITY FRACTION ALFA AND SUPP EFFECTIVNESS BETA
Y = THE INITIAL STRENGTH OF SIDE Y WITH ATTRITION COEFFICIENT A
LETHALITY FRACTION GAMMA AND SUPP EFFECTIVNESS DELTA

      COMMON X,Y,ALFA,BETA,GAMMA,DELTA,A,B,XT,YT,XE,YE,TSX,TSY,WIN,RT
  100 READ(5,9000,END=9999) X,Y,ALFA,BETA,GAMMA,DELTA,A,B,STEP
 9000 FORMAT(8F5.0,25X,F5.0 )
      WRITE(6,9005) X,B,ALFA,BETA, Y,A,GAMMA,DELTA
 9005 FORMAT('1  INITIAL FORCE PARAMETERS' / 4X,'FORCE  ATTRITION COEF
     1FIRING FOR CASULTIES   SUPPRESSIVE EFFECTIVNESS' /
     2' X ',F6.1,F14.4,F19.6,F22.4  /  ' Y ',F6.1,F14.4,F19.6,F22.4 / )
      ALH = 1.0 - STEP - 0.01
C
C     DETERMINE WHO WINS
C
      CALL TSAT
      IF ( WIN ) 3000,200,500
  200 IF  ( TSX )  295,300,305
  300 WRITE(6,9010)
 9010 FORMAT(' BOTH SIDES SUPPRESSED  ** NO COMBAT ** '  )
      GO TO 100
  295 WRITE(6,9012)
 9012 FORMAT(' BOTH ARCTANH VALUES OUT OF RANGE ')
      GO TO 100
C     BATTLE IS A TIE       ISSUE WARNING
  305 CONTINUE
      WRITE(6,9015)
 9015 FORMAT(' WARNING TSAT X = T SAT Y' / )
C
C     X WINS THE B...
C
  500 CALL STATE( TSX )
```

```
      CALL RTX
      T= RT + TSX
      WRITE(6,9025) T, XT, YT, XE, YE
 9025 FORMAT(' X WINS INITIAL COMBAT IN',F6.1,' WITH', F8.2,' SURVIVERS'
     1/ 8X,' AT T SUPPRESSED YT= ',F8.2,' AND XE YE ARE',F8.2,F7.2/)
      WRITE(6,9035)
 9035 FORMAT(6X,'ALFA',8X,'BETA',4X,'TIME',6X,'XT',7X,'YT',7X,'XE',5X,
     1 'YE' /)
C     INITILIZE VALUES
      ALFA =  0.0
      XFIX = XT
      TFIX =  T
C
C     COMPUTE BETA ALFA TRACE FOR CONSTANT CASULTIES
C
C     COMPUTE ALLOWED RANGE FOR BETA-ALFA TRACE
  600 ALFA = ALFA + STEP
      BHIGH=  Y /( (1.0 -ALFA) * B * X    )
      BLOW = 0.0
  700 BETA = (BHIGH + BLOW) / 2.0
      CALL TSAT
      IF ( WIN ) 805,750,755
  750 IF ( TSX ) 770,760,755
  760 IF ( (BHIGH - BLOW ) .GE. 0.025 ) GO TO 795
      GO TO 755
  770 IF( (BHIGH - BLOW) .GE. 100.0 ) GO TO 805
      WRITE(6,9040) ALFA,BETA
 9040 FORMAT(6X,F5.3,2X,F9.1,'   OUT OF RANGE ARCTANH')
      IF ( ALFA .LE. ALH ) GO TO 600
      GO TO 1500
  755 CALL STATE(TSX)
      IF ( ABS( XFIX - XT ) .LE. 0.025 ) GO TO 1005
      IF ( ( BHIGH - BLOW ) .LE. 0.025 ) GO TO 1000
      IF ( XFIX - XT ) 795,1005,805
  795 BHIGH = BETA
      GO TO 700
  805 BLOW =  BETA
      GO TO 700
 1000 WRITE(6,9045)
 9045 FORMAT(80X,'CAS -   TRAPPED BY BETA')
 1005 CALL RTX
      T  =  RT +  TSX
      WRITE(6,9055) ALFA,BETA,T,XT,YT,XE,YE
 9055 FORMAT(' CAS   ',F5.3,2X,F9.1 ,2X,F6.1,3X, 2(F6.2,2X,F6.2,4X) )
      IF (ALFA .LE  ALH) GO TO 600
C
C     COMPUTE BETA ALFA TRACE FOR CONSTANT TIME
C
```

```
 1500 ALFA = Y / (B * X * TFIX)
      BETA = Y/( (1.0 -ALFA)* B * X )
      TEMP = 0.0
      CALL STATE(TEMP)
      T = TFIX
      WRITE(6,9065) ALFA,BETA,T,XT,YT,XE,YE
 2000 ALFA = ALFA + STEP
      BHIGH =  Y/((1.0 -ALFA) * B * X )
      BLOW  = 0.0
 2100 BETA = (BHIGH + BLOW) / 2.0
      CALL TSAT
      IF ( WIN ) 2500,2200,2400
 2200 IF ( TSX ) 2300,2350,2400
 2300 IF( (BHIGH - BLOW) .GE. 100.0) GO TO 2500
      WRITE(6,9040) ALFA,BETA
      IF ( ALFA .LE. ALH ) GO TO 2000
      GO TO 100
 2350 IF ( ( BHIGH - BLOW ) .GE. 0.025 ) GO TU 2600
 2400 CALL STATE( TSX )
      CALL RTX
      T  = RT  + TSX
      IF ( ABS( TFIX - T ) .LE. 0.025) GO TO 2705
      IF ( (BHIGH - BLOW ) .LE. 0.025) GO TO 2700
      IF ( TFIX - T ) 2500,2705,2600
 2500 BLOW = BETA
      GO TO 2100
 2600 BHIGH = BETA
      GO TO 2100
 2700 WRITE(6,9075)
 9075 FORMAT(80X,' TIME - TRAPPED BY BETA' )
 2705 WRITE(6,9065) ALFA,BETA,T,XT,YT,XE,YE
 9065 FORMAT(' TIME ',F5.3,2X,F9.1 ,2X,F6.1,3X, 2(F6.2,2X,F6.2,4X) )
      IF (ALFA .LE. ALH) GO TO 2000
      GO TO 100
C
C     Y WINS BATTLE
C
 3000 CALL STATE( TSY )
      CALL RTY
      T = RT + TSY
      WRITE(6,9125) T,YT,XT,XE,YE
 9125 FORMAT(' Y WINS INITIAL COMBAT IN',F6.1,' WITH',F8.2,' SURVIVERS'
     1/ 8X,'AT T SUPPRESSED XT = ',F8.2,' AND XE YE ARE', 2F8.2 )
      WRITE(6, 9035)
C     INITILIZE VALUES
      ALFA = 0.0
      YFIX  = YT
      TFIX  =  T
```

```
C
C      COMPUTE BETA ALFA TRACE FOR CONSTANT TIME
C
 4000 ALFA = ALFA + STEP
      BHIGH = Y /( (1.0 - ALFA ) * B * X )
      BLOW  = 0.0
 4100 BETA = (BHIGH + BLOW) / 2.0
      CALL TSAT
      IF (WIN)  4400,4200,4800
 4200 IF (TSY)  4300,4350,4400
 4300 IF( (BHIGH - BLOW) .GE. 100.0 ) GO TO 4500
      WRITE(6,5040) ALFA,BETA
      IF ( ALFA .LE. ALH ) GO TO 4000
      ALFA = 0.0
      GO TO 5000
 4350 IF ( ( BHIGH - BLOW ) .GE. 0.025 ) GO TO 4500
 4400 CALL STATE( TSY )
      CALL RTY
      T  = RT + TSY
      IF ( ABS( TFIX - T ) .LE. 0.025 ) GO TO 4705
      IF ( ( BHIGH - BLOW ) .LE. 0.025 ) GO TO 4700
      IF ( TFIX - T) 4500,4705,4600
 4500 BHIGH = BETA
      GO TO 4100
 4600 BLOW = BETA
      GO TO 4100
C     VERIFY THAT Y CAN WIN AT THIS ALFA LEVEL
 4800 BHIGH = BETA
      BETA = 0.0
      CALL TSAT
      IF ( WIN ) 4100,4200,4000
 4700 WRITE(6,9075)
 4705 WRITE(6,9065) ALFA,BETA,T,XT,YT,XE,YE
      IF (ALFA .LE. ALH) GO TO 4000
C
C      COMPUTE BETA ALFA TRACE FOR CONSTANT CASULTIES
C
      ALFA = 0.0
 5000 ALFA = ALFA + STEP
      BLOW = 0.0
      BHIGH = Y /( (1.0 - ALFA) * B * X )
 5100 BETA = ( BHIGH + BLOW ) / 2.0
      CALL TSAT
      IF (WIN)  5400,5200,5800
 5200 IF (TSY)  5300,5350,5400
 5300 IF( (BHIGH - BLOW) .GE. 100.0 ) GO TO 5500
      WRITE(6,9040) ALFA,BETA
      IF ( ALFA .LE. ALH ) GO TO 5000
```

```
      GO TO 100
 5350 IF ( ( BHIGH - BLOW ) .GE. 0.025 ) GO TO 5500
 5400 CALL STATE(TSY)
      IF ( ABS( YFIX - YT ) .LE. 0.025 ) GO TO 5705
      IF ( (BHIGH - BLOW ) .LE. 0.025 ) GO TO 5700
      IF ( YFIX - YT )    5600,5705,5500
 5500 BHIGH = BETA
      GO TO 5100
 5600 BLOW = BETA
      GO TO 5100
 5700 WRITE(6,9045)
 5705 CALL RTY
      T = TSY + RT
      WRITE(6,9055) ALFA, BETA,T,XT,YT,XE,YE
      IF (ALFA .LE. ALH ) GO TO 5000
      GO TO 100
C     VERIFY THAT Y CAN WIN AT THIS ALFA LEVEL
 5800  BHIGH = BETA
      BETA = 0.0
      CALL TSAT
      IF (WIN)  5100,5200,5000
 9999 STOP
      END
```

SUBROUTINE STATE DETERMINES THE SURVIVING FORCES XT,YT AND THE EFFECTIVE FORCES YE AND XE AT TIME T

```
      SUBROUTINE  STATE( T )
      COMMON X,Y,ALFA,BETA,GAMMA,DELTA,A,B,XT,YT,XE,YE,TSX,TSY,WIN
      FPT = ((1.0 - ALFA)*GAMMA*BETA) + ((1.0 - GAMMA)*ALFA*DELTA)
      FMT = ((1.0 - ALFA)*GAMMA*BETA) - ((1.0 - GAMMA)*ALFA*DELTA)
      CEXT=  0.5 * A * B * FPT
      TT  = SQRT( ((A*B*FMT/2.0) **2) + (A * B * GAMMA * ALFA) )
      CHT = COSH( TT * T )
      SHT = SINH( TT * T )
      CXYT  =  EXP( CEXT * T)
      SLXT = ( ( A* GAMMA * Y/X )  + (0.5 * A * B * ( -FMT)) ) /  TT
      SLYT = ( ( B* ALFA  * X/Y ) +  (0.5 * A * B * ( FMT )) ) /  TT
      XT  = X *  CXYT  *  ( CHT  - SLXT * SHT)
      YT   = Y  *  CXYT  * ( CHT  -  SLYT * SHT )
      XE   =  ALFA  * (XT  - (1.0 - GAMMA)* DELTA * A * YT )
      YE   =  GAMMA * ( YT  - (1.0 - ALFA )* BETA  * B * XT )
      RETURN
      END
```

```
SUBROUTINE TSAT DETERMINES THE TIME AT WHICH EITHER X (TSX) OR Y (TSY)
COMPLETLY SUPPRESSES THE OPPOSING FORCE.IT ALSO DETERMINES THE WINNER
AND PASSES THE RESULT IN WIN (+1 = X, -1 = Y, 0 = TIE)

      SUBROUTINE TSAT
C
C     WIN = 0 TSX = TSY = 0 *** NO COMBAT *** (BETA TO HIGH)
C     WIN = 0  TSX = TSY = -100  BOTH ARCTANH ARG OUT OF RANGE
C     WIN = 0  TSX = TSY  .GT. 0  TIE
C
      COMMON X,Y,ALFA,BETA,GAMMA,DELTA,A,B,XT,YT,XE,YE,TSX,TSY,WIN
      FMT = ((1.0 - ALFA)* GAMMA* BETA) - (( 1.0-GAMMA)*ALFA*DELTA)
      TT  = SQRT( ((A*B*FMT/2.0) **2) + (A * B * GAMMA * ALFA) )
      SLXT = ( ( A * GAMMA * Y/X) + (0.5 * A * B * (-FMT))  ) / TT
      SLYT = ( ( B * ALFA  * X/Y) + (0.5 * A * B *   FMT )  ) / TT
      NUMX =  Y -  (1.0 -  ALFA) * BETA * B * X
      NUMY =  X -  (1.0 - GAMMA) * DELTA * A * Y
      VPX = NUMX/ ( Y * SLYT -( (1.0 - ALFA) * BETA  * B * X * SLXT ) )
      VPY = NUMY/ ( X * SLXT -( (1.0 -GAMMA) * DELTA * A * Y * SLYT ) )
      IF (NUMX .GE. 0.0) GO TO 3000
      IF ( NUMY  .GE. 0.0 ) GO TO 2500
      WIN = 0.0
      TSX = 0.0
      TSY = 0.0
      GO TO 8000
 2500 WIN = 1.0
      TSX  = 0.0
      GO TO 8000
 3000 IF  (NUMY .GE. 0.0)  GO  TO  4000
      WIN =    - 1.0
      TSY =      0.0
      GO TO 8000
 4000 IF ( (VPX  .GT. 0.0 ) .AND. ( VPX .LT. 1.0) ) GO TO 4500
      IF  ( ( VPY .GT. 0.0) .AND. (VPY .LT. 1.0) )  GO TO 5500
      WIN = 0.0
      TSX =-100.0
      TSY =-100.0
      GO TO 8000
 4500 IF ( ( VPY .GT. 0.0 ) .AND.  ( VPY .LT. 1.0) )  GO TO 6500
C
C     X WINS  WIN = +1  TSX = TIME Y SUPPRESSED
C
 4750 WIN = 1.0
      TSX = ATANH(VPX) /TT
      GO TO 8000
```

```
C
C     Y WINS   WIN = -1   TSY = TIME X SUPPRESSED
C
 5500 WIN  =   -1.0
      TSY = ATANH (VPY) / TT
      GO TO 8000
 6500 IF  ( VPX - VPY )    4750,6750,5500
C
C     TIE  $$FORCES FIGHT CN TO = SUPPRESSION TIME   WIN = 0
C
 6750 WIN = 0
      TSX = ATANH( VPX)/TT
      TSY = TSX
 8000 CONTINUE
      RETURN
      END
```

SUBROUTINES RTX,RTY COMPUTE THE TIME IT TAKES WINNER X OR Y TO COMPLETE DESTRUCTION OF LCSERS FORCES AFTER TOTAL SUPPRESSION HAS BEEN ACHIEVED.

```
      SUBROUTINE RTX
      COMMON X,Y,ALFA,BETA,GAMMA,DELTA,A,B,XT,YT,XE,YE,TSX,TSY,WIN ,RT
      RT = YT / ( ALFA * B * XT )
      RETURN
      END

      SUBROUTINE RTY
      COMMON X,Y,ALFA,BETA,GAMMA,DELTA,A,B,XT,YT,XE,YE,TSX,TSY,WIN ,RT
      RT = XT / ( A * GAMMA * YT )
      RETURN
      END


      FUNCTION ATANH(X)
      ATANH = 0.5 * ALOG ( (X + 1.0 ) / ( 1.0 - X ) )
      RETURN
      END
```

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